Price Discrimination and Market Structure:

Platform Intermediation in Vertical Oligopoly

by

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Declaration

I, Ricardo Saez Ferez, hereby declare that the work presented in this dissertation is my own original work. Where information has been derived from other sources, I confirm that this has been clearly and fully identified and acknowledged. No part of this dissertation contains material previously submitted to the examiners of this or any other university, or any material previously submitted for any other assessment.

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Date: 20/09/2023

Classification

This piece of research is primarily:

· the examination of a theoretical problem

· an analytical survey of theoretical literature

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Abstract

Quality under-provision often arises in vertical oligopolies if firms are restricted in the quality levels they can supply. When price discrimination reduces these inefficiencies, the number of firms which co-exist in the market can shrink. We investigate whether better quality provision through price discrimination can outweigh the consumer surplus losses from a less competitive environment. Our results characterise for which market structures can more discrimination be beneficial, and so speak to policy debates on whether to limit discrimination that involves sophisticated information technologies.

Acknowledgements

I dedicate the thesis to the memory of Alex Roig. I would like to like to thank Prof. Armstrong for being so generous in sharing his time. I feel grateful for my parents support, Ana & Pablo's inspiration and Elliott & Laura's friendship.

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I. Introduction

There's a long history to price-discrimination, where prices for 'similar' products differ among individuals in a way that doesn't fully reflect cost. Pigou (1920) terms as first-degree discrimination when a monopolist charges each individual his valuation or willingness to pay. All individuals purchase the good - gains from trade are realised - yet a uniform price would leave some consumers better off. While first-degree discrimination was traditionally seen as a thought experiment, advances in information technologies are fuelling an interest in reconsidering what types of discrimination are feasible, and if so, if they are desirable.

In a influential paper Bergemann, Brooks and Morris (2015), henceforth BBM, study how additional information about consumer valuations conditions the discrimination a monopolist can practice. To do so, they conceive an intermediary that has perfect information about consumer valuations. Their intermediary acts as a 'data broker'; it selectively communicates information to a monopolist that uses this to set discriminatory prices. For example, Google could share its information with, say, Apple. Alternatively, we will study an intermediary that acts as a 'platform'; it uses information, without sharing it, to set discriminatory prices for the monopolist. For example, Apple could allow Amazon to set targeted discounts for its products.

The platform conceptualisation allows for outcomes that the supplier can't commit to if it held the information. Imagine a phone shop in a small village where richer (poorer) individuals are willing to pay a lot (little) for the only phone model. Without knowledge about incomes, the supplier charges a uniform price - poor individuals can't buy. If a data broker reveals who the poor are, the supplier will charge the poor exactly their valuation. Alternatively, the supplier could sell through a platform, where he sets a uniform price. If the supplier let's the platform set personalised prices only below the uniform, as the supplier doesn't directly set prices to the poor, all gains from trade can be given to consumers.

Perhaps surprisingly, BBM prove a data broker intermediary can often realise all gains from trade so that these are fully channeled to consumers. BBM, however, limit their inquiry to the case of monopoly. We'll use the fact that conceiving intermediation through a platform is conceptually simpler to work in a oligopolistic setting. With multiple firms, interactions between discrimination and market structure can arise. In particular, more discrimination can limit the number of firms which can co-exist within a market. Our objective is to characterise the extent to which laxer competition bounds discriminations' possibilities to increase consumer welfare.

We present a model were firms are limited in the qualities they can produce and compete in a vertically differentiated oligopoly. With Mussa-Rosen (1978) preferences inefficiencies arise as lower income individuals are consume lower quality goods. A platform that reduces inefficiencies by selling high quality goods at a personalised price to poorer individuals can leave low-quality firms with no demand, altering market structure. As lower quality firms prevent high quality ones from setting higher prices to richer consumers, we ask: can gains from better quality consumption exceed the losses of a less competitive environment ?

Section II reviews BBM's results on the limits of discrimination under monopoly. We show that a data broker and platform achieve the same outcomes with different distributional consequences. Section III surveys Haghpanah and Siegel's (2023) extension of BBM's results to a Mussa-Rosen (1978) monopolistic setting with vertical (linear) preferences and nonlinear costs. Section IV selectively reviews discrimination under oligopoly. Rhodes and Zhou (2022) provide a general model of first-degree discrimination, and Elliott et al. (2021) further extends BBM's results. We leave aside information design to present a simple model of platform discrimination in Section V. We first compare welfare outcomes of a monopolistic market with discrimination to that of a duopolistic market without discrimination. Section VI develops more exhaustively the case of Bertrand competition, while Section VII extends the result to Cournot competition and includes nonlinear costs. Results mainly take the form of numerical simulations, comparing market structures under different specifications. As a way of conclusion, Section VIII describes limitations and further directions in which the work could be extended, such as endogenising for quality choice.

II. Linear Monopoly

The price-discrimination literature has been extensively surveyed; in the context of monopoly (Varian, 1989), oligopoly (Stole, 2006; Armstrong, 2006) and with a focus on information technology (Fudenberg and Villas-Boas, 2012). There are many recent information technology inspired papers, such as Hidir and Vellodi (2018) and Ichihashi (2020). This new wave mainly stems from BBM, from which we'll eventually take an unusual turn by moving away from the 'information design' methodology (Bergemann and Morris, 2019). This section therefore explains the particularities and our point of departure from BBM, leaving to subsequent sections the review of other relevant results.

BBM's article pertains to the literature that considers the outcomes (eg. consumer surplus) produced by monopoly third-degree discrimination - setting different prices to groups of individuals (i.e. segments) rather than a uniform aggregate market price. First-degree discrimination is a special case of this; each individual is in itself a segment. A classic result, due to Robinson (1933), concerns identifying for which shapes of segment demand curves does total surplus increase with discrimination. Intuitively, concave (convex) demand curves imply price changes have a small (large) impact on quantity. In most cases, discrimination increases (decreases) prices in high (low) demand segments; so total surplus increases if high demand segments have relatively less concave demands.

Aguirre et al. (2010) derive more precise sufficient conditions on the outcomes produced by particular segmentations. They assume profit functions for a market segment are strictly concave; as prices decrease in a market, the marginal gain from selling to one more consumer won't, at some point, compensate from the loses from lowering prices to infra-marginal consumers. In BBM this regularity condition can be locally violated; for instance, low valuation consumers are mixed with high valuation ones such that the monopolist is indifferent between offering the uniform price and the lowest valuation as a price. This allows BBM to obtain sufficient conditions on more general types of segmentations.

To justify the existence of such segmentations BBM invoke the concept of a 'data broker' intermediary. If segmentations are exogenous, only a sub-set of all segmentations may be of interest - as in Aguirre et al. (2010). A classic exogenous segmentation is younger individuals usually being more sensitive to price. Yet information technologies allow for data collection opportunities (eg. bank transaction data) that could approximate how much each individual values every good. Segmentations are then endogenous if an intermediary uses this data to group individuals as he wishes. Now, rather than having, say, age groups with different demands, the exact shape of the demand curve for every group is the intermediary's instrument of design. Essentially the intermediary communicates noisy signals about individuals' valuation by mixing them into segments; firms then choose prices knowing what individuals are in what groups.

With information intermediation it becomes natural to ask which combinations of consumer and producer surplus can arise for all possible segmentations (i.e. every way to combine individuals into groups). BBM conceptualise a particular segmentation as a way to structure information. Thus, third-degree discrimination is all that lies in between knowing (A) the prior distribution of types in an aggregate market to (B) every individual reservation price. Point A (B) leads to a uniform price (first-degree discrimination), with a surplus distribution as in Figure 1 (2). Figure 3 plots A and B's associated consumer and producer surplus, along with the following bounds (i) voluntary participation; a consumer won't pay more than their valuation (ii) profitable information; producers obtain at least the surplus from no segmentation uniform pricing, (iii) efficiency frontier; total surplus can't exceed the sum of all valuations.

Figure 1

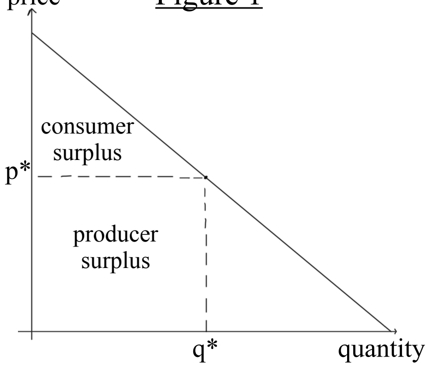


Figure 2

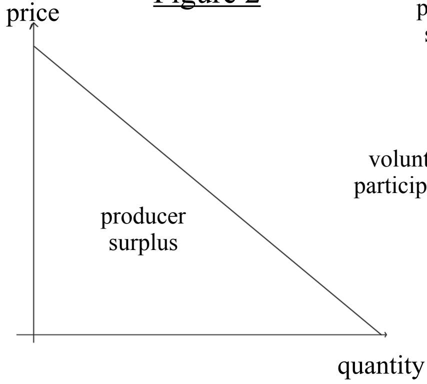
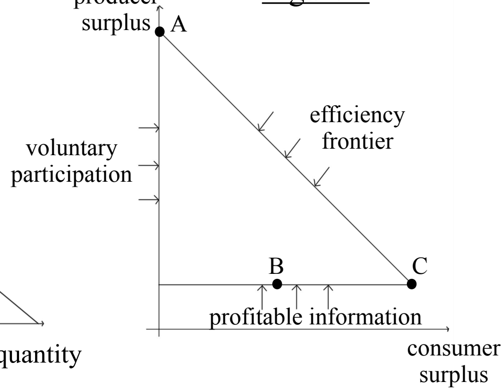


Figure 3



One of the main results of BBM is that every point inside Figure's 3 bounds can be attained by some information structure. Thus, a data broker can split efficiency gains relative to uniform pricing in many ways between consumers and producers. Point C, or first-best consumer, is of particular interest as it transfers consumers all gains from discrimination. BBM explain how to attain it when cost of production are normalised to zero. We illustrate this when valuations are given by $v = [3,3,3,2,2,2,1,1]$, an element being the valuation of an individual. The optimal uniform price is $p^u = 2$, consumer surplus $s^c = 3$, producer surplus $s^p = 12$ and deadweight losses $s^l = 2$.

To attain C, individuals with the lowest valuation must go into the same group, $g^1 = [1,1]$. The optimal price for the segment is $p^1 = 1$ so, as all individuals are served, we are in the efficiency frontier, yet consumer surplus is zero. To redistribute surplus, for each higher valuation group, $v^3 = [3,3,3]$ and $v^2 = [2,2,2]$, a share must enter g^1. In particular, the same share of v^3 and v^2 must go into g^1, so their relative proportion remains equal to that of the aggregate prior distribution. With the share equal to zero $p^1 = 1$, but if the share equaled one, so $g^1 = v$, then $p^1 = p^u = 2$. BBM show there exists a share such that the producer is indifferent between charging $p^1 = 1$ and $p^1 = p^u = 2$. In our example, this share is $1/3$, so that $g^1 = [3,2,1,1]$. As the residual consumers not in g^1 are in the same relative proportion as in the original population, we can iteratively apply this same process to the residual segments. Doing so we arrive to, $g = (g^1, g^2) = ([3,2,1,1], [3,3,2,2])$, $p = [p^1, p^2] = [1,2]$, $s^c = 5$, $s^p = 12$ and $s^l = 0$.

The data broker can choose how to split gains between producers and consumers such that, for all $\lambda = [0,1]$, $s^c = 3 + 2\lambda$ and $s^p = 12 + 2(1 - \lambda)$. For a lower λ the intermediary simply, for instance, changes to $g = (g^1, g^2) = ([3,3,3,2,2,2], [1,1])$; so that $s^c = 3$, $s^p = 14$ and $s^l = 0$. This merely shows the supplier will use all additional information about valuations to extract consumer surplus. What is remarkable is BBM prove third-degree discrimination can achieve the same total surplus as first-degree and distribute all surplus to consumers. The data broker intermediary prevents surplus extraction by structuring its information. Yet, this might not be the only way through which intermediation can result in gains for both consumers and producers. As an alternative, we develop the concept of a 'platform' intermediary, which is analytically simpler.

It is convenient to think of our intermediary as a platform much alike Amazon - it knowing everyone's valuations. In Amazon, suppliers set a uniform price for a product. We then ask: what if the supplier allows Amazon to charge individuals lower prices than the uniform? If the supplier is a monopolist, the platform could selectively offer discounts (i.e. personalise prices) to consumers that would otherwise be unserved - those with valuations below the uniform. In some way, the supplier delegates its price setting strategy to the platform; the platform can then distribute part of the surplus gains from discrimination to the supplier.

More precisely, the platform can charge the originally unserved consumers exactly their valuation (what the firm would do if it had this information) or anything higher than marginal cost - thereby also choosing λ. In other words, it can achieve every point within the bounds of Figure 3 without communicating information - we simplify the problem away from 'information design'.

For the platform to increase consumer surplus, we wan't to assume the uniform price the monopolist sets is the same as if no intermediation existed. From the consumer side, this seems natural as otherwise consumers wouldn't reveal information nor use the platform. A potential concern is: can the monopolist use the platform to its advantage by charing higher uniform prices ? Notice the producer doesn't gain information through using the platform; if the platform sells only to the unserved consumers, the optimal price for the originally served is the same. The problem arises as the producer could raise prices above the uniform to let the platform sell to some of the originally served consumers.

Assume the platform's regular practice is to set prices to the unserved such that gains from discrimination are split equally between consumers and producers, i.e. $\lambda = 0.5$. Consider $v = [7,4,1]$, $p^u = 4$, $s^c = 3$, $s^p = 8$, $s^l = 1$. If the platform follows $\lambda = 0.5$, the monopolist's optimal response is to increase $p^u = 7$. Notice $g = (g^m, g^p) = ([7], [4,1])$, $p = [p^u, p^p] = [7,[2,0.5]]$, were p^p are personalised prices and g^s (g^p) is the group served by the monopolist (platform), so $s^c = 2.5$, $s^p = 9.5$, $s^l = 0$, leads to larger profits than $g = (g^m, g^p) = ([4,2], [1])$, $p = [p^u, p^p] = [4,[0.5]]$, $s^c = 3.5$, $s^p = 8.5$, $s^l = 0$. To prevent the latter outcome, which decreases consumer surplus, the platform can apply a 'threat strategy'. That is, the platform can set $p = [p^u, p^p] = [7,[0,0]]$, so $s^c = 5$, $s^p = 7$, $s^l = 0$, so that it is in the interest of the monopolist to maintain $p^u = 4$. If the uniform price is profit maximising, the gains from increasing prices to infra-marginal consumers are always lower than the losses from not selling to a marginal consumer. Thus the platform has the bargaining power to prevent the monopolist from capturing surplus from infra-marginal consumers.

While a data broker and platform intermediary can attain the same points in the efficiency frontier, they distribute surplus between individuals differently. Consider the first-best consumer outcome the data broker and platform attains with $g = ([3,2,1,1], [3,3,2,2])$, $p = [1,2]$ and $g = ([3,3,3,2,2,2], [1,1])$, $p = [2,[0,0]]$ respectively. For both we have that $s^c = 5$, $s^p = 12$ and $s^l = 0$. With a data broker the surplus gains from discrimination go to higher valuation consumers which are randomly grouped with lower valuations ones in g^1 - as Figure 4 shows. Figure 5 is the residual market g^1. Reducing inefficiencies from low valuation individuals being unserved implies higher valuation ones gain information rents. With a platform surplus gains are channeled to the lower valuation individuals, as shown in Figure 6. The monopolist can't extract this surplus itself, so it's indifferent in giving it to consumers. With purely vertical preferences, distributional consequences might be of interest.

Figure 4

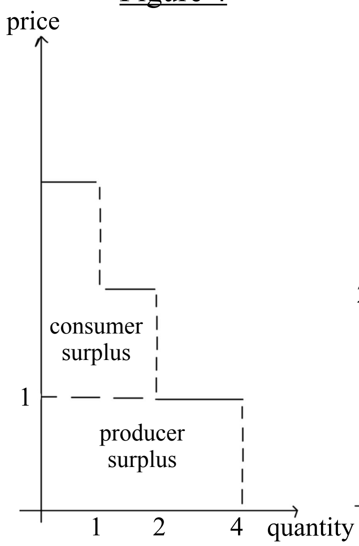


Figure 5

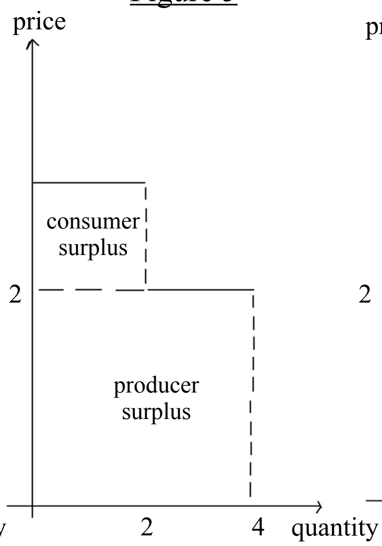
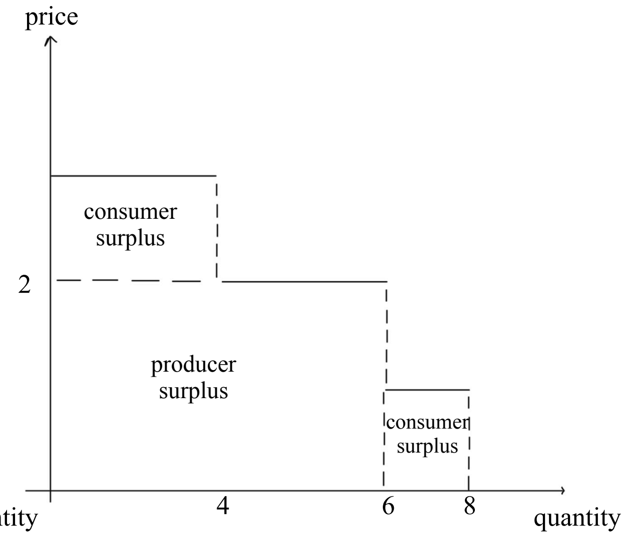


Figure 6

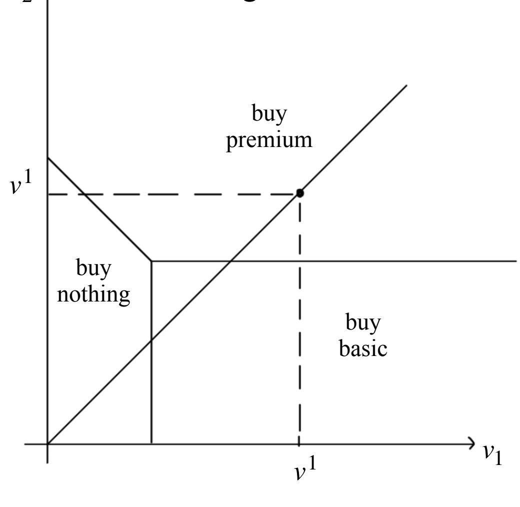


III. Nonlinear Monopoly

The previous section implicitly assumes consumers demand a single unit of a unique good. Yet if valuations are nonlinear in the goods' quality (or quantity), the monopolist can find it optimal to offers a range of price-quality menus. This is known as second-degree discrimination; the monopolist exploits the information revealed through consumer self-selection, but the menus of lower types must be distorted to prevent high types from choosing them. BBM document their results' robustness with nonlinear valuations borrowing the preferences in Maskin and Riley (1984). Instead we study the equivalent setting of (linear) Mussa-Rosen (1978) preferences with nonlinear costs. The extension of BBM's results to such setting is provided by Haghpanah and Siegel's (2023), which we'll review below. We first provide a simple account of second-degree discriminations following the 'upgrades' approach closely following Armstrong's (2016) survey.

Consider a consumer willing to pay v_1 for a basic product and $v_1 + v_2$ for a premium one. The producer charges price p_1 for a unit of the basic product, incurring a cost of c_1, and $p_1 + p_2$ for the premium one, incurring a cost of $c_1 + c_2$. When $v_2 = k\,v_1$ we have Mussa-Rosen (1978) preferences; consumers have the same intrinsic preferences but differ in their income - being richer increases valuations equiproportionately. For simplicity we assume $v_1 = v_2 = v$, i.e. $k = 1$. The consumer chooses out of 3 discrete choice options the one that results in a highest net surplus. The consumer buys the premium product if $2v - (p_1 + p_2) > max(v - p_1, 0)$, the basic product if $v - p_1 > max(2v - (p_1 + p_2), 0)$ and nothing otherwise. Figure 7 borrows Armstrong's (2016) representation of demand patterns, including a consumer with valuation v^1 $(= v_1^1 = v_2^1)$ which lies in the line satisfying Mussa-Rosen preferences.

Figure 7



When $c_1 = c_2 = 0$, Stokey (1979) proves it's optimal for the monopolist not to second-degree discriminate. In other words, the monopolist maximises its profit by setting prices (note $p_1 = p_2 = p$) such that only the premium product is bought, as shown graphically in Figure 8. Two outcomes can arise, depending on the distribution of consumer preferences. In Figure 9 valuations are 'similar', eg. $v = [4,4.1]$, so that the optimal price schedule maximises consumer and total surplus. In Figure 10 valuations are 'dis-similar', eg. $v = [1,4]$, which minimises consumer surplus and total surplus. In the latter case, lowering p_1 to serve the poor would imply giving up surplus from the rich - to prevent the latter switching to the basic good.

Figure 8

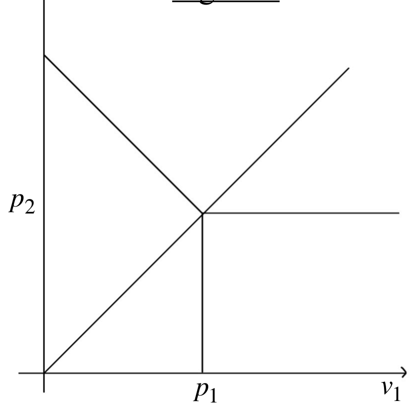


Figure 9

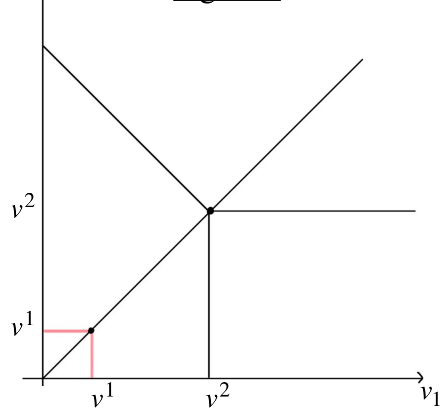
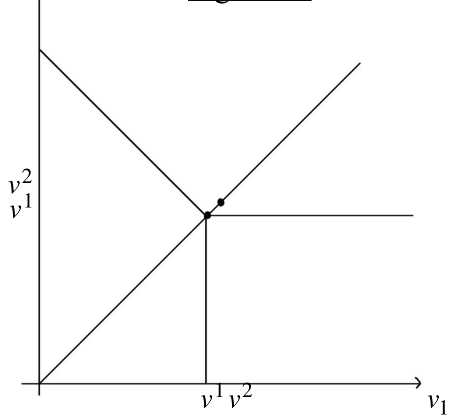


Figure 10



A data broker could eliminate the inefficiencies (red lines) from Figure 10. We extend the numerical example from Section I, to include non-linear valuations. Note that as $v_1 = v_2 = v$ we can abuse notation to express $v = [3,3,3,2,2,2,1,1]$ as before. Again we can abuse notation as $p_1 = p_2 = p$, so that $p^u = 2$ as before. A market segmentation such that consumers gain all surplus is $g = [[3,3,3,2,2], [2,1,1]]$, $p = [2,1]$, which differs from that in Section I. Haghpanah and Siegel's (2023) prove that if inefficiencies are caused by the seller excluding some consumers, then first-best consumer can be achieved via segmentation as in BBM. A platform intermediary would achieve this with $g = (g^s, g^{us}) = ([3,3,3,2,2,2], [1,1])$, $p = [2,[0,0]]$; the analysis is trivial, results follow from the previous section. Now the data broker distributes gains from discrimination to the richer individuals, while the platform to poorer ones.

To allow for second-degree discrimination let $c_1 = 1$ and $c_2 = 2$. If the monopolist supplies the poorer with the basic goods it gives up surplus from the richer but faces a lower cost of supply. To see this let $v = [5,3]$ and assume $p = 3$, which is true for $c_1 = c_2 = 0$. We plot this in Figure 11, the allocation is efficient, but the monopolist forgoes surplus from the high type (the green lines). To capture the surplus, the monopolist serves the basic good to the poor by raising the price for the 'upgrade', p_2, to the point were the richer is indifferent to switching to the basic. Figure 12 shows how the net surplus received from serving the poorer consumer (red line) doesn't compensate for the extra surplus extracted from the richer one (fragment of blue line previously green). Besides inefficiencies from lower quality consumption, the poorer individuals pay more ($1/3 > 1/4$) per unit of quality than the richer. Note if $k = 3$, as show in Figure 13 and 14, the extra cost of supplying the premium product isn't steeper than consumer valuations (so-called 'increasing returns to quality' as $c_2 < kc_1$), so we're back to Stokey's (1979).

Figure 11

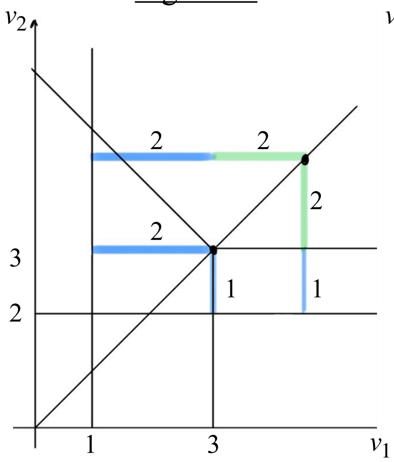


Figure 12

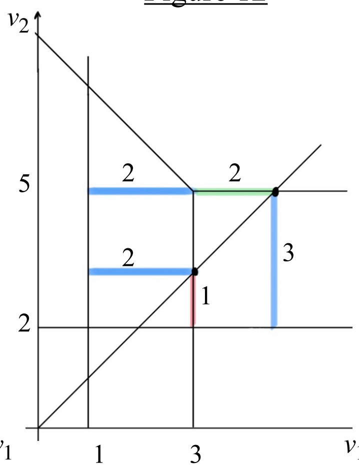


Figure 13

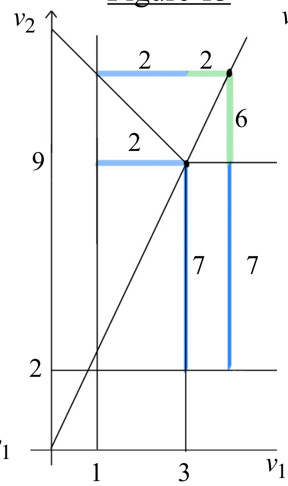
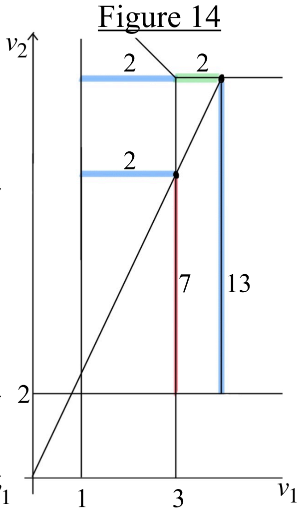


Figure 14



Can discrimination eliminate the inefficiencies (the red line) from Figure 12 such that consumers receive all gains? In this case Haghpanah and Siegel (2023) prove that BBM's results must necessarily deviate away from the first-best consumer. Let's consider $v = [5,5,3,3,3]$; with $c_1 = 1$ and $c_2 = 2$, $p^u = [p^1, p^2] = [3,5]$, $s^c = 4$, $s^p = 16$, $s^l = 3$. To reduce inefficiencies, we can segment $g = [[5],[5,3,3,3]]$, so that $p = [[5,5],[3,3]]$, $s^c = 4$, $s^p = 19$, $s^l = 0$. For there to be no second-degree discrimination in the low-type dominated market, the residual market is left with no low-types, so that the monopolist now extracts all of the surplus from the high type. Whenever there exist second-degree discrimination in the original market, the seller appropriates at least some of the surplus gains from discrimination.

A platform intermediary can simply offer the premium product at a discount to the poorer individuals. Unlike in Section II, serving lower types inevitably reveals information to the monopolist, i.e. the optimal price to the originally served increases. With second-degree discrimination, having to serve the low types is what prevents surplus extraction from the high types. If the platform reduces the inefficiencies from Figure 12, the monopolist doesn't have an incentive anymore to maintain $p_1 = 3$, so will set $p_1 = 5$. If the platform receives $p^u = [p_1, p_2] = [5,5]$, it can still enforce its bargaining power by threatening to charge $p^p = [0,0]$ to the low types. As before, there always exist a 'threat strategy' that prevents the monopolist from abusing delegation. If its optimal for the monopolist to serve a basic product, the surplus it gains from it must be higher than that he can gain from only selling the premium version. So, if the monopolist increase prices to the high type, the intermediary can always punish the monopolist by eliminating all profits from the low type. From our introductory example, if Amazon observes that when it discounts an iPhone X, the prices of iPhone X+1 increase, it will have to threat Apple by setting larger discounts for the iPhone X - which is a slightly more unrealistic situation than in Section I.

IV. Oligopoly

With price discrimination many of the intuitions developed under monopoly are overturned in oligopoly settings. While deviating to discrimination might be profitable for a single firm, if competing firms' best-response is to also discriminate, the entire industry profits can fall. That discrimination can amplify competition is best seen in Thisse and Vives's (1988) model of duopoly with firms locating in an extreme of a Hoteling line and uniformly distributed consumers. In equilibrium firms capture half the market; a deviation to lower prices isn't optimal as gains from capturing a marginal consumer (business stealing) don't compensate lowering prices to current consumers (infra-marginal loses). If firms personalise prices (i.e. first-degree discriminate) they compete for each consumer individually - there are no infra-marginal loses. When a firm sets a price of zero to the consumer furthest away from it, the other firm can always offer a non-zero price that leaves that consumer with a (epsilon) higher net utility. These indifference prices are higher for non-central consumers with stronger preferences. The result is that every personalised price is strictly below the uniform, central consumer facing the lowest prices.

Advances in information technologies motivate Rhodes and Zhou (2022) to generalise Thisse and Vives's (1988) model. They consider more general (eg. normal) distributions of consumers along the hoteling line. With more consumers located centrally, a price decrease will capture more consumers than before, leading to lower equilibrium uniform prices. Now first-degree discrimination raises some prices but lowers others, leading to ambiguous welfare effects. In addition, their model allows for the market to be uncovered; personalised prices allows serving some consumers that didn't purchase with uniform prices - increasing consumer surplus. Interestingly, uncovered markets imply that those purchasing under uniform prices have stronger preferences over one type - for which discrimination doesn't intensify competition much. In this general model, it can be the case that both consumer and producer surplus increases with discrimination, or either can increase at the expense of the other.

First-degree discrimination is only one information structure (full information); other segmentations within the efficiency frontier can be of interest if, for instance, regulators desire surplus combinations that benefit both producers and consumers. This motivates Elliott et al. (2021) to extend BBM's results to oligopoly when a data broker communicates the same information to all firms. Now some segmentations amplify competition more than others, as business stealing gains are relatively larger than infra-marginal losses. To see this, define v_1^1 (v_2^1) as individual 1's valuation for firm's 1 (2) product. Consider a segment that mixes individuals with 'similar' preferred products, $g = [[v_1^1, v_2^1], \ldots, [v_1^n, v_2^n]] = [[6,3], [2,1]]$, so $p^g = [p_1^g, p_2^g] = [2,0]$, which incentivises firms to compete more aggressively for every individual. Mixing individuals with the same preferred product doesn't always amplify competition, such as when $g = [[6,1], [2,1]]$, $p^g = [5,1]$. Competition is more often completely suppressed if segments mix individuals with 'dis-similar' preferred products, such as when $g = [[6,1], [1,6]]$, $p^g = [6,6]$, but this isn't always the case, as when $g = [[2,1], [1,2]]$, $p^g = [1,1]$. Elliott et al. (2021) results are analogous to BBM; they show which consumer-optimal and producer-optimal information structures can be constructed by combining these insights.

Alike in the previous sections, conceiving the intermediary as a platform proves to be analytically simpler. It's natural to assume the platform must increase both consumer and producer surplus for it to be used - which occurs only if markets are uncovered. The analysis in the platform case is trivial; unserved consumers can be simply offered a personalised discount only from the firm that is their preferred option - avoiding the need to consider competitive reactions. This surplus can be divided between producers and consumers in as many ways as desired. We can apply the same arguments as before, to craft a 'threat strategy' (profit maximising behaviour implies surplus obtained through the platform is necessarily lower than that already enjoyed) and show distributional considerations apply as in the monopoly case.

V. Duopoly Versus Monopoly

To make our analysis non-trivial, we leverage from the conceptual simplicity of platform intermediation to consider how more discrimination interacts with market structure in the case of pure vertical oligopoly. With two firms, a high (low) quality supplier, henceforth 'top' ('bottom'), sells to those individuals with high (low) taste for quality - valuations being fully pinned down by income. When valuations are above marginal costs, efficiency implies all consumers purchase the top good. That is, total surplus increases if a platform sells the top good at a personalised discounted price to consumers that would buy the bottom good. This, however, can result in a less competitive environment (eg. monopoly rather than duopoly) if the platform leaves the bottom supplier with a zero demand. If so, the top firm will sets higher prices (profit increases); yet, some consumers gain from better quality consumption - this surplus can be split between suppliers, consumers and the platform in many ways. Assuming all gains from better quality consumption are given to consumers, we aim to quantify if this can be enough to offset losses from a laxer competitive environment.

Corts (1998) studies a similar problem and setting; market equilibrium entails a bottom firm selling to consumers that only shop on price ('cheap') and a top one to those with a preference for quality ('choosy'). The author proves the ability of the top firm to price discriminate more, by targeting cheap consumers with sales, incentivises the bottom firm to go for aggressive price cuts for the choosy consumers. This amplification of competition can lead to lower profits for both firms - a strategic commitment not to price discriminate is profitable. The technology we consider, thought, is more sophisticated than sales - the platform sets personalised prices with perfect knowledge. The policy the platform would apply to set discriminatory prices if both firms used it is unclear; as the bottom firms' product is inferior the platform can always drive its demand to zero. We assume the platform aims to eliminate inefficiencies created by the bottom firm, so it will only sell its services to the top firm.

If the platform targets just a fraction of those otherwise consuming the bottom good, the bottom firm would, as in Corts (1998), set lower prices - infra-marginal losses become smaller. We can assume the platform could also target discounts precisely to the consumers the bottom firm aims to attract with price cuts. Knowing this the low quality firm won't fight for the top's firms consumers, which prevents the escalation of competition. To limit the platforms' abilities to reduce inefficiencies, the platform must capture all of the bottom's firms market. With an arbitrarily small fixed cost, if the bottom firm faces zero demand it will exit or not enter the market; more discrimination relaxes competition. Now 'threat strategies' seem less plausible; if the market structure is a monopoly, the platform can't force the surviving firm to set its prices as in the duopoly situation. If this were the case, the monopoly wouldn't use the platform. A data-broker faces the same challenge; revealing all consumer valuations leaves the bottom firm with zero demand - the top firm would then set prices as a monopolist.

To get a flavour of the analysis that concerns us we consider the simplest possible scenario and progressively build up from it. We borrow the model of pure vertical oligopoly from Tirole (1986, pg. 296) . As in Mussa and Rosen (1978) consumers are indexed by their willingness to pay for quality θ, which is uniformly distributed across the population between $\underline{\theta}$ and $\bar{\theta} = \underline{\theta} + 1$, density being 1. Individual θ gains utility $U = \theta s - p$ only for the first unit it consumes of quality s, and is left with zero utility otherwise. There are two firms, firm i supplies a good of quality s_i, where $s_2 > s_1$. We assume s_i is exogenously given (eg. supplier s_2 has won the latest patent race) and normalise constant marginal cost of production to zero for all firms.

Assuming $\bar{\theta} > 2\underline{\theta}$, there is sufficient consumer heterogeneity for both firms to face positive demand. As a consumer buys only if $\theta p_i > s_i$, the market is covered when $\underline{\theta} p_1 > s_1$ - this is always he case if $\frac{\bar{\theta} - 2\underline{\theta}}{3}(s_2 - s_1) < \underline{\theta} s_1$, which we assume. A consumer with taste θ purchases from supplier 2 instead of 1 when $\theta s_2 - p_2 > \theta s_1 - p_1$. Thereby, the consumer θ_1 is indifferent between both firms exactly when $\theta_1 = \frac{p_2 - p_1}{s_2 - s_1}$. With a uniform distribution, demand for firm 2 (1) is simply the consumer mass that lies above (below) θ_1. With price competition (i.e. Nash-Bertrand game), profit for the bottom and top supplier is $\pi_1(p_1, p_2) = (\theta_1 - \underline{\theta})p_1$ and $\pi_2(p_1, p_2) = (\bar{\theta} - \theta_1)p_2$ respectively. It can be shown that equilibrium prices are given by $p_1 = \frac{\bar{\theta} - 2\underline{\theta}}{3}(s_2 - s_1)$ and $p_2 = \frac{2\bar{\theta} - \underline{\theta}}{3}(s_2 - s_1)$; derivations are provided in the next section.

The market structure that prevails entails positive profits for both firms. To see this, notice that the top firm can always set a price, p_2^0, so that if $p_1^0 = 0$ the bottom firm faces zero demand. Maintaining $p_1^0 = 0$, the top firm raises its price to $p_2^* > p_2^0$, were infra-marginal gains are equal to the losses from a marginal consumer switching to the bottom firm. At p_2^* a mass, x_1, switches to the bottom firm, which has an incentive to raise price to $p_1^* > p_1^0 > 0$, were infra-marginal and switching forces equilibrate. At this price, a mass x_2 ($< x_1$) switches back to the top firm. Now p_2^* isn't an equilibrium; infra-marginal gains from a larger mass of consumers are greater. The top firm raises again its price, a mass switches to the bottom; the bottom raises its price, and so on. Iterating the process the mass of consumers switching becomes arbitrarily small, at which neither firm finds it profitable to deviate. The top firm is initially restricted in the market power it exerts; when it leaves a residual mass of consumers to the bottom, the latter exercises its more limited market power to this mass, augmenting the market power of the top.

The question we are after is simple: can consumer surplus be larger when a monopoly discriminates through a platform than in duopoly without discrimination? Deriving optimal prices for the monopolist is straightforward. Recall a consumer buys only if $\theta p > s$, thereby the consumer $\theta_0 = \frac{p}{s}$ is indifferent between buying or not. Profit for the monopolist is simply $\pi_m(p_m) = (\bar{\theta} - \theta_0)p_m$, so that $p_m = \frac{\bar{\theta}s}{2}$. When marginal costs are zero, we know the monopolist doesn't benefit from second-degree discriminating (i.e. selling a good of less quality than s_2) from Stokey (1979). A platform discriminates for the monopolist by selling at a discount to those otherwise not buying and/or that would consume from a lower quality firm. Of particular interest is the case when the platform sets a price of zero, as consumer surplus is maximised - if these prices don't leave consumers better off, others won't either.

A simple numerical example shows that consumer gains from more discrimination cane be bounded by competition. Let $s_2 = 1$, $s_2 = 2$, $\underline{\theta} = 0.5$ and $\bar{\theta} = 1.5$. Using the solutions above $p_1 = 0.16, p_2 = 0.83, \theta_1 = 0.66, p_m = 1.5, \theta_0 = 0.75$ so demand for firm 1, 2 and the monopolist is $d_1 = 0.16$, $d_2 = 0.84$ and $d_m = 0.75$ respectively. Notice the monopolist has a lower demand than in the duopoly situation ($d_m < d_2$); it is reasonable to assume the platform serves to the mass the monopolist would sell under duopoly ($\theta_0 - \theta_1$). Consumer surplus under monopoly and duopoly is given by $C_m = \int_{\underline{\theta}}^{\bar{\theta}} s_2\theta \, d\theta \; - (\bar{\theta} - \theta_0)p_m$ and $C_d = \int_{\underline{\theta}}^{\theta_1} (s_1\theta - p_1)d\theta + \int_{\theta_1}^{\bar{\theta}} (s_2\theta - p_2)d\theta$ respectively. For our example, we have $C_m = 1.18$ and $C_d = 0.875$. As $C_d > C_m$ we conclude that the platform can't increase both consumer and producer surplus if the bottom firm faces zero demand.

Going beyond a numerical example requires understanding how altering the quality gap, i.e. $s_2 - s_1$, affects firms ability to exert market power. Consider first $s_2 - s_1 < \epsilon$, if we increase s_2 consumers are more heterogeneous - the difference in valuations for the top good between two adjacent individuals is larger. A price increase results in a lower mass of consumers switching to the competing firm. When balancing infra-marginal and switching forces firms raise their prices more than before - profits increase for both firms as seen in Figure 15. This is Shacked and Sutton's (1982) principle of maximal differentiation. Figure 16 shows that for an uncovered market, as studied in Section VI, for $s_2 - s_1 > \epsilon$ an increase in s_1 can raise profits for the bottom firm, as it serves a larger consumer mass. At some point, however, the effect of firms competing more aggressively - the difference in valuation for the top good relative to the bottom shrinks - dominates.

Figure 15

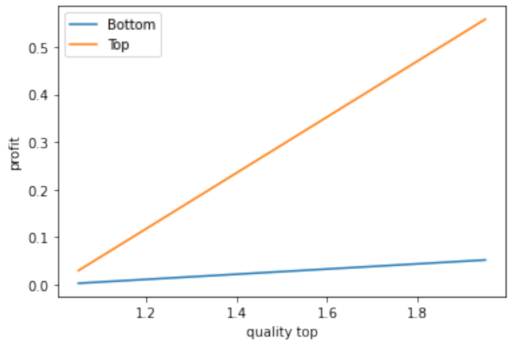
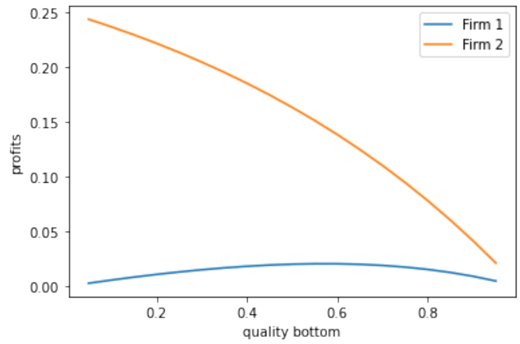


Figure 16



Intuitively, the monopoly market structure with discrimination results in consumers being better off than in a duopoly only for a large enough $s_2 - s_1$, when the bottom firm's competitive pressure is weak. More precisely, in appendix 1 we show that for $\underline{\theta} = 0.9$, $C_d > C_m$ only if $\frac{1739}{1800}s_1 > \frac{113}{1200}s_2$. To understand the result, Figure 17 plots market primitives. Notice that a higher s_1 shifts valuations linearly, so market shares remain equal. With duopoly a higher s_1 pushes the top firm away from the monopoly prices - both firms enjoy less market power. We can further decompose results as in Figure 18, showing the consumer surplus generated by eg. the bottom firm in duopoly (i.e. $\int_{\underline{\theta}}^{\theta_1} (s_1\theta - p_1)d\theta$). As A high s_1 implies there is a large difference in the welfare the top firm generates when it is a monopolist rather than competing in duopoly; gains from better quality consumption due to the platform don't compensate.

Figure 17

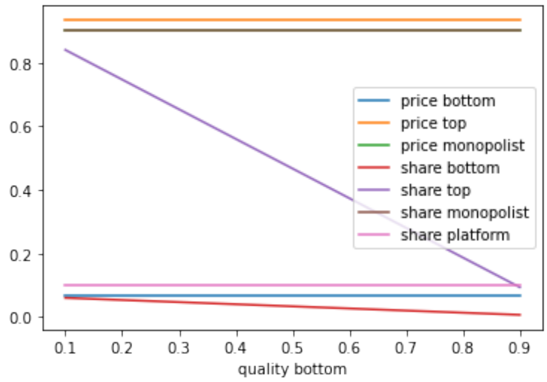
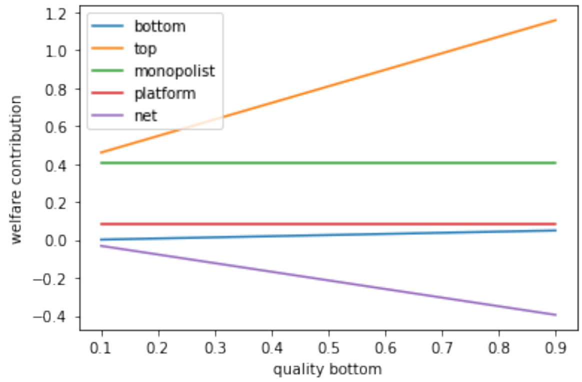


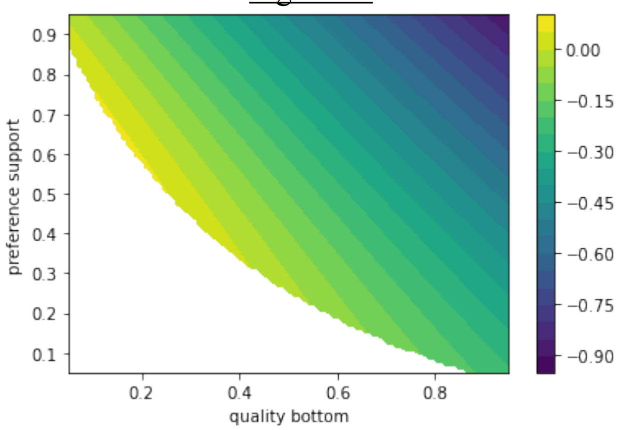
Figure 18



To extend our analysis to different $[\underline{\theta}, \bar{\theta}]$ distributions, notice that a smaller $\underline{\theta}$ means consumers are more 'dis-similar'; although the range of the interval $[\underline{\theta}, \bar{\theta}]$ is always one, the relative difference between 3 and 4 is smaller than that between 0 and 1 (Gabszewicz et al., 1986). More consumer heterogeneity allows the bottom firm to exercise more market power - the top firm has to reduce its prices more if it want's to attract the same mass of consumers. So, consumer benefits the most from competition when they are homogeneous (i.e. high $\underline{\theta}$); for which we expect a comparison between duopoly to monopoly to be more harmful for consumers. To test these hypothesis we can calculate $C_s^d - C_s^m$ for all combinations of s_2, s_1 and $\underline{\theta}$ (recall $\bar{\theta} = \underline{\theta} + 1$). In Appendix 2, we show that $C_s^d - C_s^m > 0$, only if $s_1 > s_2 \dfrac{13 - 2\underline{\theta} - 11\underline{\theta}^2}{22 + 16\underline{\theta} - 2\underline{\theta}^2}$.

Related to the analytical result, Figure 19 plots a simulation were we fix $s_2 = 1$ and consider all combinations of values such that $s_1 < 1$ and $\underline{\theta} < 1$. Note that the regularity assumptions that ensure a covered market and positive market shares always holds - some areas are blank. The Python code for all of the simulations that we run is included in Appendix 3. Our simulation confirms the intuition that for any s_1 the harm to consumers from there not being a bottom firm are greater when there is less heterogeneity from both the firm and consumer dimensions.

Figure 19



Before studying more intricate market structures, we justify the ensuing analysis can be done similarly to that in this section. To allow for many firms to co-exist in Bertrand equilibrium and the existence of Cournot equilibrium, the next section considers an uncovered market. Now if the platform captures the entire demand the bottom firm can face, the bottom firm can set lower prices to attract consumers previously unserved. In the case of zero marginal costs, it is reasonable to assume the platform can also capture all of uncovered market. Yet, with non-linear marginal costs, the bottom firm will always be able to sell to an arbitrary small number of consumers which the top firm can only serve at a loss. We assume the bottom firm doesn't cover its fixed cost when it can't serve the mass of consumers it would do without the platform - so it won't enter/exit the market. At last, with more than two firms, the platform could sell to a share of the next-to-bottom (intermediate quality) ones. If so, the intermediate firm would have the incentive to lower prices to fight for the bottoms' consumers. In the limit, this also results in the bottom firm not entering/exiting the market. It is analytically simpler to consider only the case were the platform targets its discounts to the bottoms' consumers - we'll stick to this in the following sections.

VI. Bertrand

We extend our model following Gabszewicz et al. (2016). First, by allowing for n firms $i = 1,2,...,n$ to offer n quality variants, each firm selling a single variant of the product, were $s_n > s_{n-1} > \ldots > s_1 > 0$. Second, we let taste for quality, θ, be uniformly distributed in the interval $[0,\bar{\theta}]$ so that the market remains uncovered. At last, the constant marginal costs of firm i is given by c_i, were $c_n > c_{n-1} > \ldots > c_1 > 0$ so that cost increasing with quality. Recall $\theta_0 = \dfrac{p_1}{v_1}$ indexes the consumer indifferent between buying the bottom quality and not purchasing, and consumer θ_i indexes the consumer indifferent between $i+1$ and i were $\theta_i(p_i, p_{p+1}) = \dfrac{p_{p+1} - p_i}{s_{i+1} - s_i}$. When suppliers compete in prices, profit for the bottom, every intermediate and top supplier are given by $\pi_1(p_1, p_2) = (\theta_1 - \theta_0)(p_1 - c_1)$, $\pi_i(p_{i-1}, p_i, p_{i+1}) = (\theta_i - \theta_{i-1})(p_i - c_i)$ and $\pi_n(p_1, p_2) = (\bar{\theta} - \theta_{n-1})(p_n - c_n)$ respectively. Gabszewicz et al. (2016) prove a Nash equilibrium price vector is guaranteed for any finite number of firms.

We can express the previous in Matrix Algebra notation, by letting

$$B = \begin{bmatrix} -\frac{1}{s_2 - s_1} - \frac{1}{s_1} & \frac{1}{s_2 - s_1} & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \frac{1}{s_2 - s_1} & -\frac{1}{s_3 - s_2} - \frac{1}{s_2 - s_1} & \frac{1}{s_3 - s_2} & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & \frac{1}{s_3 - s_2} & -\frac{1}{s_4 - s_3} - \frac{1}{s_3 - s_2} & \frac{1}{s_4 - s_3} & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & \cdot & \cdot & 0 & \frac{1}{s_i - s_{i-1}} & -\frac{1}{s_{i+1} - s_i} - \frac{1}{s_i - s_{i-1}} & \frac{1}{s_{i+1} - s_i} & 0 & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \frac{1}{s_n - s_{n-1}} & -\frac{1}{s_n - s_{n-1}} \end{bmatrix}$$

$$A = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \bar{\theta} \end{bmatrix}, p = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix}, q = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix}, c = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} \text{so demand is } q = A + Bp.$$

$$\text{Define } Q = \begin{bmatrix} q_1 & 0 & \cdot & 0 \\ 0 & q_2 & 0 & 0 \\ 0 & \cdot & \cdot & 0 \\ 0 & \cdot & 0 & q_n \end{bmatrix}, \text{ so } \begin{bmatrix} \pi_1 \\ \pi_2 \\ \vdots \\ \pi_n \end{bmatrix} = \begin{bmatrix} q_1 & 0 & \cdot & 0 \\ 0 & q_2 & 0 & 0 \\ 0 & \cdot & \cdot & 0 \\ 0 & \cdot & 0 & q_n \end{bmatrix} \begin{bmatrix} p_1 - c_1 \\ p_2 - c_2 \\ \vdots \\ p_n - c_n \end{bmatrix} = Q(p - c)$$

For expository purposes let $N = 3$ so demand is:

Profits for every individual firm are given by:

$$\begin{bmatrix}\pi_1\\ \pi_2\\ \pi_3\end{bmatrix} = \begin{bmatrix}q_1 & 0 & 0\\ 0 & q_2 & 0\\ 0 & 0 & q_3\end{bmatrix}\begin{bmatrix}p_1-c_1\\ p_2-c_2\\ p_3-c_3\end{bmatrix} = \begin{bmatrix}0\\ 0\\ \bar{\theta}p_3\end{bmatrix} + \begin{bmatrix}(p_1-c_1)(-\frac{1}{s_2-s_1}-\frac{1}{s_1}) & (p_1-c_1)\frac{1}{s_2-s_1} & 0\\ (p_2-c_2)\frac{1}{s_2-s_1} & (p_2-c_2)(-\frac{1}{s_3-s_2}-\frac{1}{s_2-s_1}) & (p_2-c_2)\frac{1}{s_3-s_2}\\ 0 & (p_3-c_3)\frac{1}{s_3-s_2} & (p_3-c_3)(-\frac{1}{s_3-s_2})\end{bmatrix}\begin{bmatrix}p_1\\ p_2\\ p_3\end{bmatrix}$$

Define as B_1 to the matrix B with its diagonal elements multiplied by 2, and C_1 to vector c with c_i multiplied by the the i'th diagonal element of B. Taking the first order conditions we obtain:

$$A + B_1 p - C_1 = \begin{bmatrix}0\\ 0\\ \bar{\theta}\end{bmatrix} + \begin{bmatrix}2(-\frac{1}{s_2-s_1}-\frac{1}{s_1}) & \frac{1}{s_2-s_1} & 0\\ \frac{1}{s_2-s_1} & 2(-\frac{1}{s_3-s_2}-\frac{1}{s_2-s_1}) & \frac{1}{s_3-s_2}\\ 0 & \frac{1}{s_3-s_2} & -2\frac{1}{s_3-s_2}\end{bmatrix}\begin{bmatrix}p_1\\ p_2\\ p_3\end{bmatrix} - \begin{bmatrix}c_1(-\frac{1}{s_2-s_1}-\frac{1}{s_1})\\ c_2(-\frac{1}{s_3-s_2}-\frac{1}{s_2-s_1})\\ c_3(-\frac{1}{s_3+s_2})\end{bmatrix} = 0$$

Defining $s = [s_1, s_2, \ldots, s_{n-1}, s_n]$, then the solution to the previous system depends only on s, $\bar{\theta}$ and c. Solving for equilibrium prices we obtain $p^b(s, \bar{\theta}, c) = B_1^{-1}(C_1 - A)$. We assume that $c_i = 0$ for all firms unless otherwise stated.

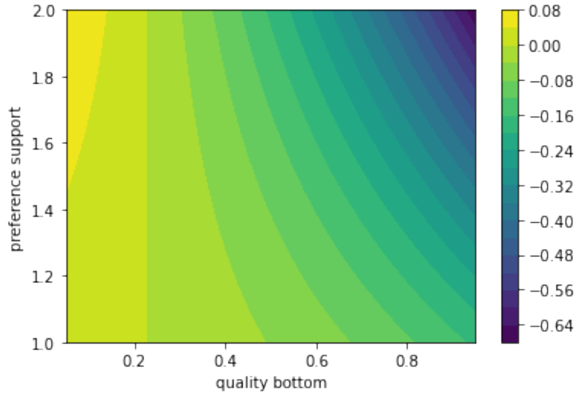
We briefly consider how the analysis from Section V changes for an uncovered market. Normalising for density and including a uncovered market, we can calculate consumer surplus for duopoly and monopoly as

$$C_d = \frac{\theta_1^d - \theta_0^d}{\bar{\theta}}\int_{\theta_0^d}^{\theta_1^d}(s_1\theta - p_1)d\theta + \frac{\bar{\theta} - \theta_1^d}{\bar{\theta}}\int_{\theta_1^d}^{\bar{\theta}}(s_2\theta - p_2)d\theta \text{, and}$$

$$C_m = \frac{\bar{\theta} - \theta_0^m}{\bar{\theta}}\int_{\theta_0^m}^{\bar{\theta}}(s_2\theta - p)\ d\theta\ + \frac{\theta_0^m - \theta_0^d}{\bar{\theta}}\int_{\theta_0^d}^{\theta_0^m} s_2\theta\ d\theta$$

respectively, slightly abusing notation by denoting θ^d (θ^m) to the indifferent consumers for duopoly (monopoly). Figure 20 plots the results for a simulation for all combinations of $\bar{\theta}$, s_2 and s_1, such that $s_2 = 1$, $1 < \bar{\theta} < 2$ and $s_1 < 1$. Consumer surplus being higher with platform intermediation is not much more common. The main difference from the previous section is that the share of consumers served by the top (bottom) firm is lower (higher) than before, in turn making the share the platform serves larger, so that its relative welfare contribution is greater. This arises from the fact that the uncovered market implicitly means consumers have more heterogeneous preferences than above (which is why we don't need regularity assumptions for both firms to have positive shares). The results are similar to Section V, so that intuitions follow and we can conclude that more preference heterogeneity can't overturn the fact that platform intermediation most likely harms consumers.

Figure 20



We want to allow for multiple firms to co-exist with discrimination, the simplest extension being a comparison between 3 firms and duopoly. The position of the intermediate firm relative to the rest adds an extra layer of interaction. When $\frac{s_3 - s_2}{s_2 - s_1} = 1$ as p_2 increases consumers switch equally to top and bottom; yet if $\frac{s_3 - s_2}{s_2 - s_1} > 1$ (< 1) the intermediate firm competes more closely with the bottom (top). We can alter the quality gap in three ways, either by increasing s_1 or s_3, or by changing the position of s_2 relative to s_1 and s_3. Figure 21, 22 and 23 plot the percentage change in profits as we increase $s_3 > 2$ for $s = [1,2,s_3]$, $2 > s_1 > 1$ for $s = [s_1,2,3]$ and $3 > s_2 > 1$ for $s = [1,s_2,3]$ respectively. As before, firms enjoy higher profits when they are more spaced from each other in the quality space. From the increase in x we see as the intermediate firm begins to compete more closely with the bottom, the top firm's profit isn't much affected. At last, all firms suffer when the intermediate competes too closely with the top.

Figure 21

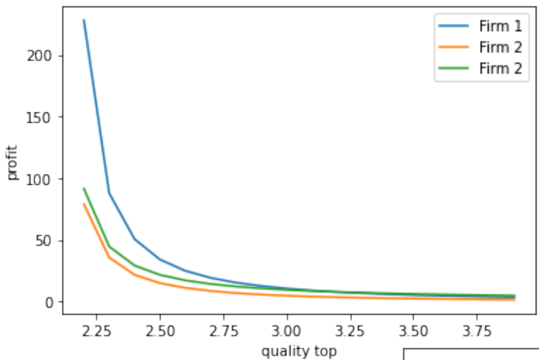


Figure 22

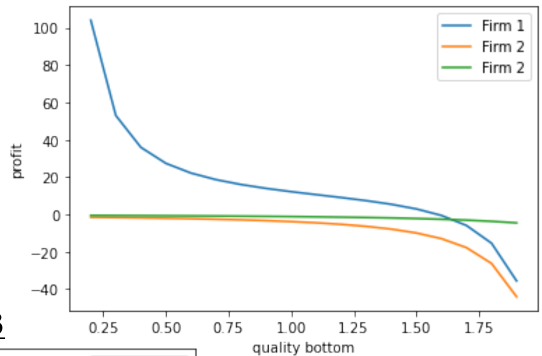
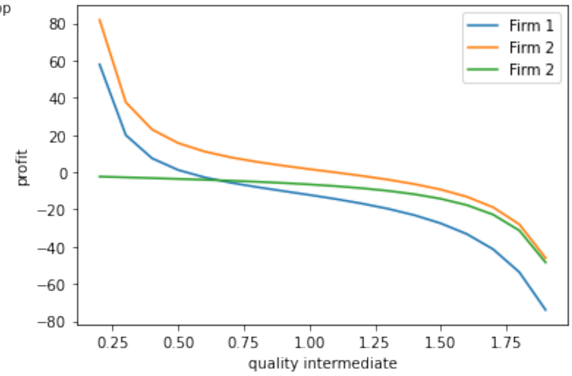


Figure 23



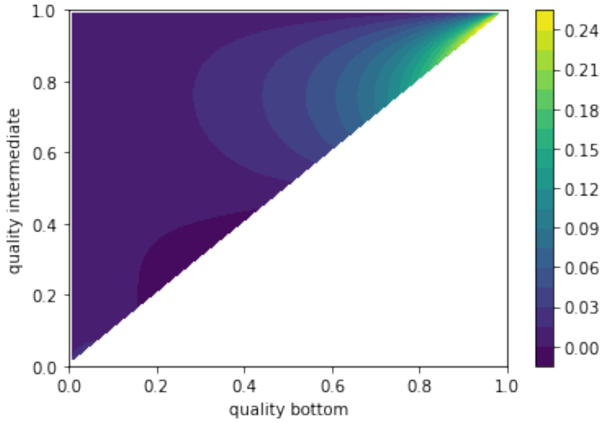
To evaluate if 2 firms and discrimination can improve 3 firms with no discrimination, let's briefly consider a numerical example when $\bar{\theta} = 1$. For $s^3 = [s_1, s_2, s_3] = [1,2,3]$, $p^b(s^3) = [p_1, p_2, p_3] = [0.04,0.15,0.57]$, and the indifferent consumers are given by $[\theta_0^3, \theta_1^3, \theta_2^3] = [0.04,0.12,0.42]$. For $s^2 = [2,3]$, $p^b(s^2) = [0.2,0.6]$, $[\theta_0^2, \theta_1^2] = [0.1,0.4]$. With 3 firms the platform aims to reduce the inefficiencies from $[0.04,0.12]$, however, some of the consumers in the range $[0.10,0.12]$ are already served in the 2-firm market structure. The simplest way to proceed is assuming that with discrimination consumers fall into the bounds defined by $[\theta_0^2, \theta_1^2]$, so the platform targets $[0.04,0.1]$. We can calculate the difference in consumer surplus between 3 firms and duopoly. More generally, defining $\Theta_i^n = \dfrac{\theta_{i+1}^n - \theta_i^n}{\bar{\theta}}$, we have that for n firms consumer surplus is given by:

$$C_n = \Theta_0^n \int_{\theta_0^n}^{\theta_1^n} (s_1\theta - p_1)d\theta + \ldots + \Theta_i^n \int_{\theta_i^n}^{\theta_{i+1}^n} (s_i\theta - p_i)d\theta + \ldots + \Theta_{n-1}^n \int_{\theta_{n-1}^n}^{\bar{\theta}} (s_n\theta - p_n)d\theta$$

This can compared to that consumer surplus of a market with n-1 firms and adding the gains from platform discrimination, i.e. $C_{n-1} + \dfrac{\theta_0^n - \theta_0^{n-1}}{\bar{\theta}} \displaystyle\int_{\theta_0^{n-1}}^{\theta_0^n} s_n\theta \; d\theta.$

We proceed by presenting general results, from which we'll try to break down the intuitions with more specific simulations. Figure 24 runs a simulation for all combinations $v = [s_1, s_2, s_3]$ that satisfy $s_1 < s_2 < s_3 < 1$ for $\bar{\theta} = 1$. Note this is equivalent to running one $s_1 < s_2 < s_3 < 2$, or $\bar{\theta} = 2$; the graphs produced are the same with different magnitudes. Figure 24 thereby contains all the results which can be obtained with our model. The main difference we observe from the duopoly/monopoly comparison is that now the platform nearly always achieves an outcome that increases consumer surplus. Intuitively, results are better than in the duopoly/monopoly case as now more discrimination doesn't completely eliminate competition in the market. More surprising is the fact that consumer surplus increases the most when all of the firms are more alike (i.e. compete more fiercely), which is the completely the opposite to the Section V.

Figure 24



To understand better results we run a simulation for $v = [s_1, 0.8, 1]$, $s_1 < 0.8$ as s_1 increases. Figure 25 plots the consumer surplus generated by, for example, the intermediate firm in the market structure with discrimination (2 firms) minus that it generates in the market structure without discrimination (3 firms); for the bottom and platform we subtract zero. The top firm mainly drives our results, rather than the platform directly. It seems counter-intuitive that the top firm creates more surplus with 2 than with 3 firms, so in Figure 26 we plot differences in primitives (eg. price for the top firm in market with 3 firms minus that for a market with 2 firms).

Figure 25

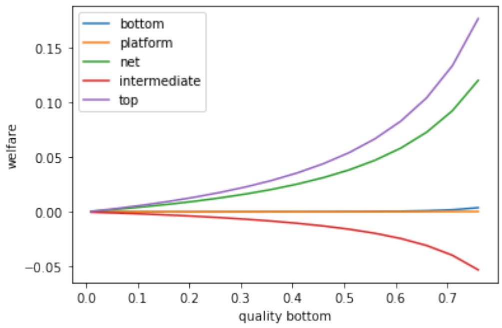
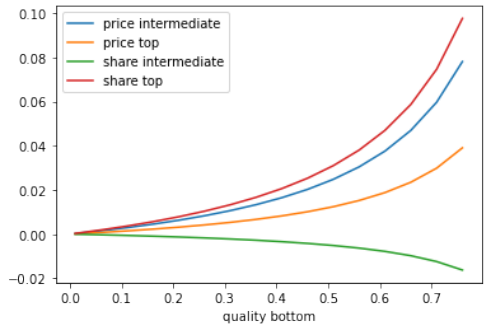


Figure 26



Notice the change in market share for the top increases with s_1. To understand why, consider the market structure with 3 firms and a low s_1. The intermediate firm competes closely with the top, yet the bottom has little influence on the other firms - so other firms react little when it ceases to be part of the market. We know from Figure 22 that when s_1 increases the intermediate firm shifts to compete more aggressively with the bottom relative to the top; so the top firms' relative competitive position improves. In practice, the bottom and intermediate decrease their price significantly; while the top retains high prices but looses market share to the rest. The higher the s_1, the more constrained the intermediate firm is, so when the bottom firm ceases to exist, the intermediate will raise its price substantially to the consumer mass it competed aggressively with the bottom. This leaves a large consumer mass to the top, that increases prices significantly less. It is the consumption of better quality goods from the intermediate mass that switches to the top what increases net welfare.

To verify this, Figure 27 and 28 plots the primitives and welfare for the case were $v = [s_1, 0.5, 1]$ for $s_1 < 0.5$ as s_1 increases. Now the increase in the share of consumers the top serves to doesn't rise as fast as price increases, so that latter outweighs the former and the net gains in welfare are relatively small. As the intermediate and top are more far away from each other, they don't compete as fiercely as the bottom ceases to exist; the intermediate raises its prices more to the consumer mass it previously competed for with the bottom. The effects of the platform seem to be small again; the platform has a larger role in alternative specifications were either (i) the platform serves to all consumers that would purchase from the bottom firm with 3 firms, or (ii) the platform serves to a fraction of those otherwise purchasing from the intermediate, such that the intermediate competes more fiercely with the bottom until it drive its demand to zero.

Figure 27

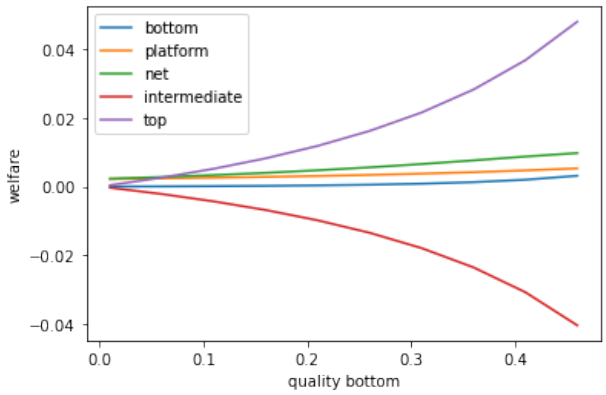
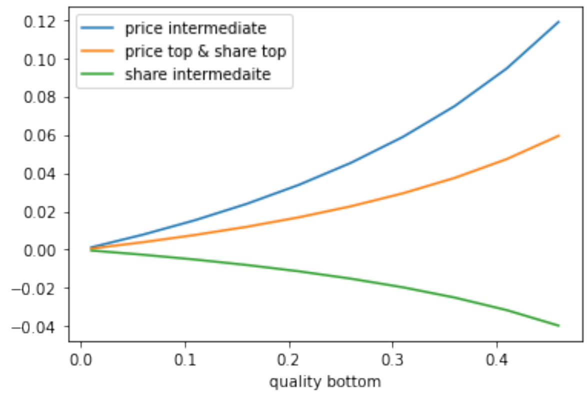


Figure 28



We can extend our results to more general market structure transitions, at a cost to interpretation. In Figure 29, 30 and 31 we plot simulations for $v = [s_1, s_2, s_3, s_4]$ fixing $s_1 = 0.1$, $s_1 = 0.5$ and $s_1 = 1$ respectively, when $s_1 < s_2 < s_3 < s_4$, $s_4 = 2$ and $\bar{\theta} = 5$. We find that consumer surplus is always greater with more discrimination, the greater the bottoms' quality the larger the effect. Additionally, the effect is larger when intermediate firms are further away from each other - for instance, $s = [1,1.5,1.6,2]$ is inferior to $s = [1,1.2,1.6,2]$. This goes well with the interpretation that if the closest competitor ceases to exist, firms start competing more fiercely with the competitor that was relatively further away from it before. That is, if the intermediate firms are head to head, i.e. $y - x < \epsilon$, the will compete fiercely regardless of the rest of the industry structure; so eliminating the bottom has little effects. However, when they are further apart, $x - y > \epsilon$, the fact the bottom isn't present significantly alters their incentives - the intermediate firms compete more closely with each other.

Figure 29

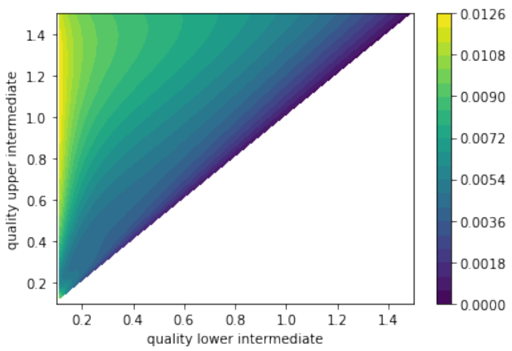


Figure 30

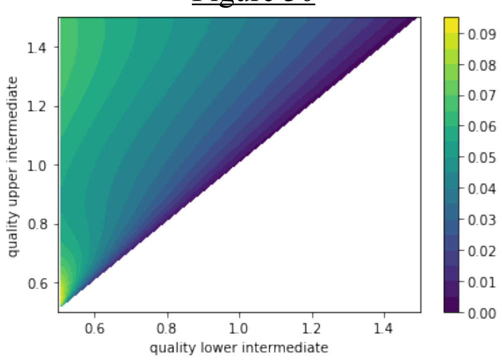
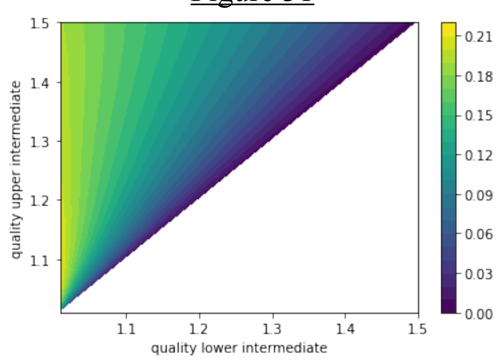


Figure 31



While Bertrand competition is an appropriate representation of many industries, it is hard to accommodate multi-product firms when competition is that fierce. Nonetheless, it is worth commenting how previous results change as $c_n > c_{n-1} > \ldots > c_1 > 0$ but there are ‘increasing returns to quality’ so that we are under Stokey’s (1979) case were second-degree discrimination isn’t optimal for firms. With positive costs a decrease in price to attract the marginal consumer implies lower net gains; firms have an incentive to set higher prices. With ‘increasing returns to quality’, the top firms competitive position is improved as it can attract, by lowering prices, marginal consumers at a relatively lower cost.

Calculating consumer surplus with costs adds mild complications. The platform must sell the top firm’s product above marginal cost, so for the monopolist situation so we have $C_m - c_2(\theta_0^m - \theta_0^d)$. In addition, it can be the case that the platform can’t serve above cost to all the consumers the bottom firm was serving to, which occurs if $\theta_1 s_2 - c_2 < 0$; so that $\theta_0^m = \dfrac{c_2}{s_2}$ for C^m. Also note that $p_m = \dfrac{1}{2}(\bar{\theta} s_2 + c_2)$. With this, we can replicate the analysis from Figure 20; that is, in Figure 32, 33 and 34 , we plot $C_m - C_d$ for different values of $\bar{\theta}$, for $c_i(s_i) = 0.1 s_i^{0.1}$, $c_i(s_i) = 0.15 s_i^{0.1}$ and $c_i(s_i) = 0.15 s_i^{0.2}$ respectively. Increasing cost has a dual effect, it increases the prices the platform sets but the reduces the competitive power of the bottom firm (it not being part of the market being less harmful). When the latter is greater enough, a positive consumer surplus when switching to a monopoly proves to be more common. In general, the greater the ‘returns to quality’ the more the gains from platform intermediation.

Figure 32

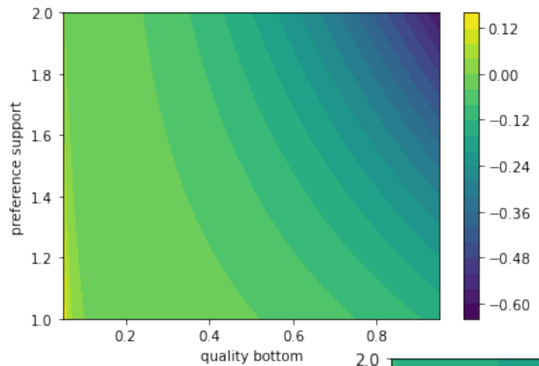


Figure 33

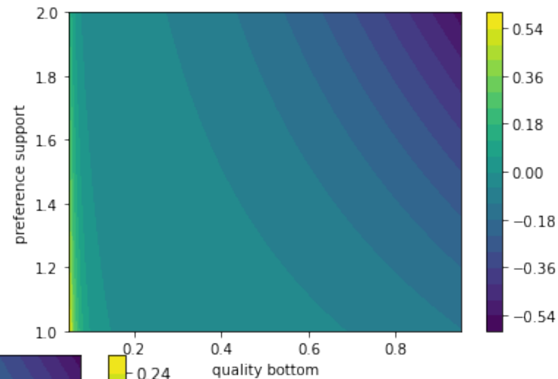
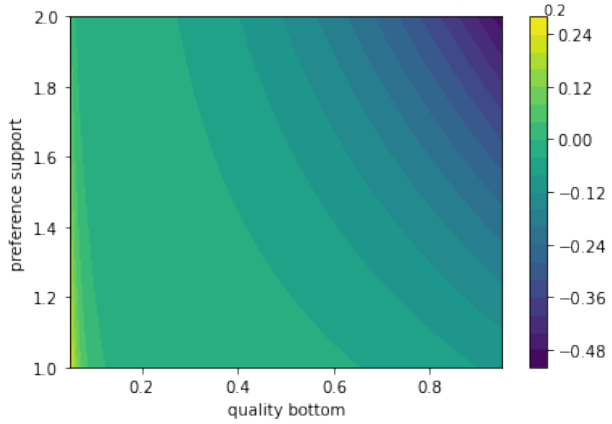


Figure 34



VII. Cournot

Before extending the model to allow for multi-product firms, we discuss how Cournot competition alters the results from the previous sections. We can derive a Cournot-Nash solution by closely following the system of equations from Section VI but with quantities as choice variables. As $q = A + Bp$, inverse demand is $p = B^{-1}(q - A)$; then $\pi = Q(B^{-1}(q - A) - c)$, and we can take first order conditions $B_1^{-1}q - B^{-1}A - c = 0$, were we define Q and B_1 as in Section VI. We find that $q = (B_1^{-1})^{-1}(B^{-1}A + c)$; substituting back, $p^c = B^{-1}((B_1^{-1})^{-1}B^{-1}(A + c)) - A$. If in Bertrand equilibrium the top (bottom) firm holds a mass of 0.8 (0.2); in a quantity setting game, the bottom firm (top) decreases quantity (i.e. increase prices by balancing infra-marginal and switching forces) so that a fraction of its 0.2 (0.8) mass becomes unserved. Cournot competition results in higher prices than Bertrand, but requires justifying that if a firm decreases the quantity it sells, this won't be met by the competitor increasing the quantity it supplies. We are merely interested on how the fact that Cournot competition is less fierce our previous results.

Recall that with Bertrand competition a larger quality gap when $s_2 - s_1 < \epsilon$ and s_2 increases resulted in greater profits for both firms. Figure 36 shows this isn't the case with Cournot competition. As s_2 increases the bottom firm serves to a smaller consumer mass, so the the infra-marginal gains from leaving consumers unserved is smaller and it increases its prices less - this effect dominates that of firms becoming more dis-similar. Instead, when $s_2 - s_1 > \epsilon$ and we increase s_1 the bottom firm always faces a higher profit - the effect of firms being more similar is dominated. Figure 37 shows that as the bottom firm is able to offer an attractive good its profit rises much more (and that of the top decreases much less) than in the Bertrand competition case, reaching levels similar to the top.

Figure 35

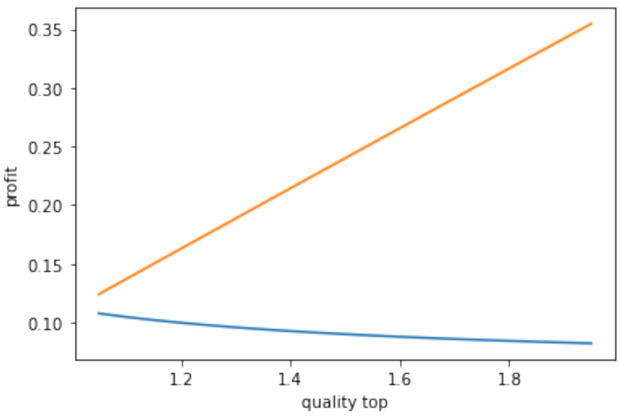
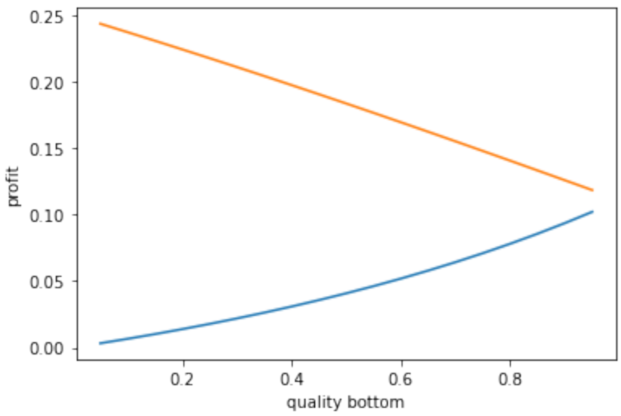


Figure 36



As Cournot competition changes the patterns of competitive interaction and relaxes competition we expect it to influence results. Figure 37, 38 and 39 runs the equivalent simulations presented in Figure 20, 24, and 30 from before; that is a comparison between monopoly and duopoly (with various $\underline{\theta}$), three firms to duopoly, and four firms to three firms, respectively. With Cournot competition the platform intermediation always increases consumer surplus - and more than in the Bertrand counterpart. With two firms and monopoly, a larger s_1 now results in both firms earning larger profits; monopoly doesn't imply drastic price increases, so the effect of the platform offsets there being a less competitive environment. Comparing 3 and 2 firms, consumer surplus gains are now larger when the bottom is further away from the intermediate and the top; if the intermediate is far from the bottom, as the latter ceases to exist, the former doesn't capture that much of the bottoms' base (i.e. the platform serves to a larger mass). At last, with 4 and 3 firms, firms being more similar to each other increases consumer surplus - the opposite to before.

Figure 37

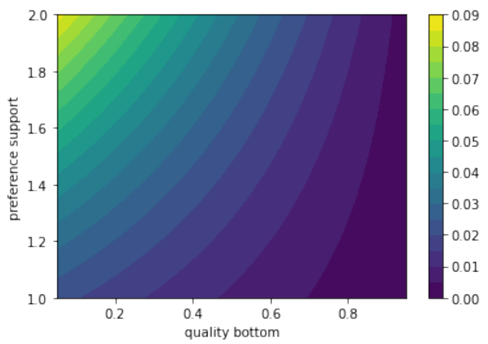


Figure 38

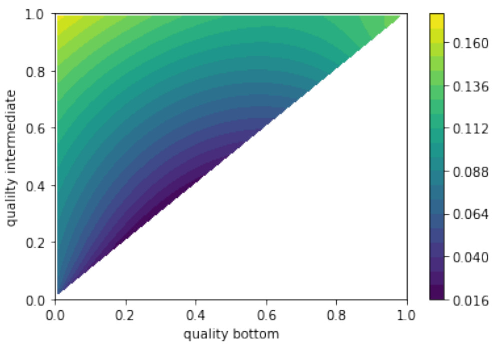
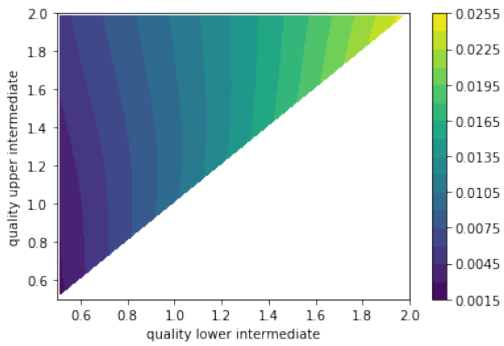


Figure 39



The aim of this section is to extend our analysis to more general cost structures, so that the top firms finds it profitable to second-degree discriminate. Recall from our discussion in Section III, that second-degree discrimination occurs only under 'decreasing returns' to quality. This is satisfied if $\frac{c_2 - c_1}{q_2 - q_1} < \ldots < \frac{c_n - c_{n-1}}{q_n - q_{n-1}}$, so we specify a function of the form $c_i(s_i) = a\,x^b$ were $a < 1$ and $b > 1$. We first consider a monopolist with 2 products and solve for its optimal product line using the notation from Section V. Profit is given by $\Pi = (A + Bp)^T(p - c)$; in its full expression:

$$\Pi_1 = (\begin{bmatrix} 0 \\ 0 \\ \bar{\theta} \end{bmatrix} + [p_1 \quad p_2] \begin{bmatrix} -\frac{1}{s_2 - s_1} - \frac{1}{s_1} & \frac{1}{s_2 - s_1} \\ \frac{1}{s_2 - s_1} & -\frac{1}{s_2 - s_1} \end{bmatrix}) \begin{bmatrix} p_1 - c_1 \\ p_2 - c_2 \end{bmatrix}$$

Taking the first order conditions we obtain:

$$A + 2Bp - Bc = \begin{bmatrix} 0 \\ 0 \\ \bar{\theta} \end{bmatrix} + \begin{bmatrix} -\frac{2}{s_2 - s_1} - \frac{2}{s_1} & \frac{2}{s_2 - s_1} \\ \frac{2}{s_2 - s_1} & -\frac{2}{s_2 - s_1} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} - \begin{bmatrix} -\frac{1}{s_2 - s_1} - \frac{1}{s_1} & \frac{1}{s_2 - s_1} \\ \frac{1}{s_2 - s_1} & -\frac{1}{s_2 - s_1} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = 0$$

So that, $p = \dfrac{1}{2}B^{-1}(Bc - A)$. If the monopolist can supply two products with a maximum quality s_2, a simple numerical algorithm can determine the optimal $s_1 < s_2$ based on $c_i(s_i)$ and $\bar{\theta}$. That is, we iterate over all combinations s_1 such that $s_1 < s_2$, compute the associated profits, and then pick the s_1 that maximises profit.

We can consider the slightly more intricate case of 2 firms, were the top firm offers 2 products, as above, and the bottom offers only one product. We can combine the first order conditions from the multi-product monopoly and those from the Bertrand competition from Section VI, arriving to:

$$A + B_2 p - C_2 = \begin{bmatrix} 0 \\ 0 \\ \bar{\theta} \end{bmatrix} + \begin{bmatrix} -\frac{2}{s_2 - s_1} - \frac{2}{s_1} & \frac{2}{s_2 - s_1} & 0 \\ \frac{2}{s_2 - s_1} & (-\frac{2}{s_3 - s_2} - \frac{2}{s_2 - s_1}) & \frac{1}{s_3 - s_2} \\ 0 & \frac{1}{s_3 - s_2} & -\frac{2}{s_3 - s_2} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} - \begin{bmatrix} c_1(-\frac{1}{s_2 - s_1} - \frac{1}{s_1}) + c_2 \frac{1}{s_2 - s_1} \\ c_2(-\frac{1}{s_3 - s_2} + \frac{1}{s_2 - s_1}) + c_1 \frac{1}{s_2 - s_1} \\ c_3(-\frac{1}{s_3 + s_2}) \end{bmatrix} = 0$$

The Bertrand solution for the multi-product duopoly is $p^b = B_2^{-1}(C_2 - A)$. Notice we have implicitly defined B_2 and C_2 as the combined first order conditions for the multi-product monopoly, $2B$, and Bertrand competition, B_1 in Section VI - likewise for C_2. We can derive the same result Cournot competition. From $\pi = Q(B^{-1}(q - A) - c)$ the first order condition is now, $B_2^{-1}q - B^{-1}A - c = 0$, with B_2 defined as above. We arrive to $q = ((B_2^{-1})^{-1}(B^{-1}A + c))$ so that $p^c = B^{-1}((B_2^{-1})^{-1}(B^{-1}A + C)) - A$.

Note that a monopolist will always find it profitable to sell two products rather than one; this isn't necessarily true in the duopoly situation - an extra product intensifies competition. If the top firm offers an extra product close to that of the bottom, the latter will react by decreasing price, which can overall harm the top firm. This is documented in what Johnson and Myatt (2003) call product line 'pruning'. Again, if the top firm can produce a maximum quality s_3, we can calculate numerically if introducing a second product s_2 is optimal, and if so, what is the profit maximising $s_2 < s_3$.

We can use the previous solution to evaluate if platform intermediation with a monopoly can improve the duopolistic market. Let's work with a numerical example. When $c_i(s_i) = 0.1x^{1.5}$, $\bar{\theta} = 2$ and $s = [1, s_2, 3]$, there is no $1 < s_2 < 3$, such that the monopolist finds it optimal to second-degree discriminate. However, with $c_i(s_i) = 0.3x^{1.5}$ the monopolist does find it optimal to introduce a intermediate quality s_2. The optimal value of

s_2 the top firm desires to introduce, is $s = [1{,}2.3{,}3]$, resulting in $c = [0.3{,}1.04{,}1.55]$. We then have, $p^b(s^1) = [0.82{,}2.56{,}3.51]$, so that $[\theta_1^2, \theta_2^2, \theta_3^2] = [0.82{,}1.34{,}1.37]$. The simplest way to proceed to assume that the monopolist only produces $s_3 = 3$, so $p_m = 3.78$ and $\theta_0 = 1.25$. As $\theta_0 s_3 - c_3 > \theta_0 s_1 - p_1$, the platform can sell to all of those that would otherwise consume the bottoms' firm product.

This, however, is not the best the monopolist can do, as it can reap higher profits by second-degree discriminating. Let's assume that, as in the duopoly structure, the monopolist can produce two products. Now the optimal product line is $s = [1.1{,}3]$, and $p_m = [p_1, p_2] = [1.1{,}3]$. Following the discussion in section III , the platform will now sell the best product, s_3, to those consuming an inferior quality (eliminating inefficiencies from second-degree discrimination) and those otherwise unserved. Consumer surplus is:

$$C_m = \Theta_0^m \int_{\theta_0^m}^{\theta_1^m} (s_2\theta - p_1)\ d\theta\ + \Theta_1^m \int_{\theta_1^m}^{\bar{\theta}} (s_2\theta - p_2)\ d\theta\ + \frac{\theta_0^m - \theta_0^d}{\bar{\theta}} \int_{\theta_0^d}^{\theta_0^m} s_2\theta\ d\theta\ - c_2(\theta_0^m - \theta_0^d)$$

We run simulations under this last specification for $s = [s_1, s_2, 1]$, $s_1 < s_2 < 1$ and allow for second-degree discrimination (i.e. top produces $s_2 > 0$) only when profitable. Figure 40, 41 and 42 plots for the cost structures $c_i(s_i) = 0.1 s_i^{1.3}$, $c_i(s_i) = 0.3 s_i^{1.3}$ and $c_i(s_i) = 0.3 s_i^{1.4}$ respectively. We find that consumer surplus remains positive but lower than in the single product cases. Intuitively, as 'decreasing returns to quality' makes the bottom firm more competitive than before, it restricts more the top's price - the platform harms consumers more. Not surprisingly, larger costs for the top firm exacerbates the previous effect. In general, we can conclude that cost structures with 'decreasing returns to quality' produce results analogous to the ones with zero costs yet with slightly less favourable outcomes for platform discrimination.

Figure 40

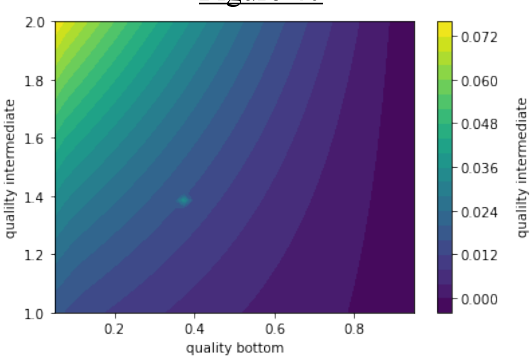


Figure 41

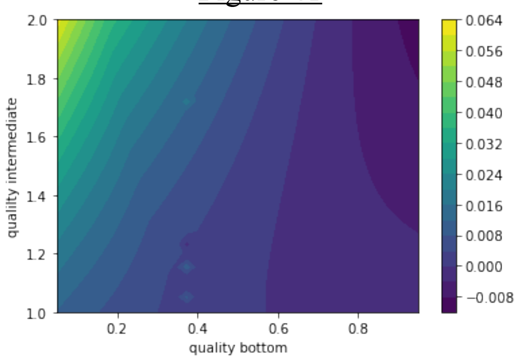
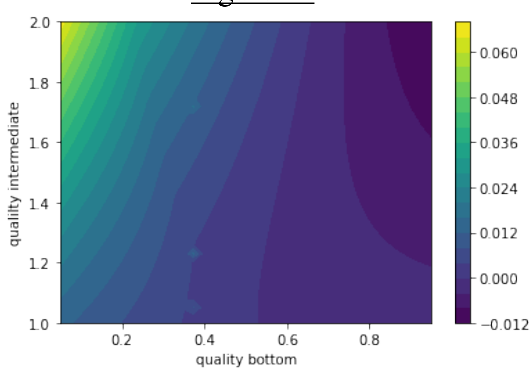


Figure 42



VIII. Conclusion

Our aim was to study how an intermediary with perfect valuation information interferes with market structure in a purely vertical oligopoly. We largely simplified our enquiry by, instead of working with 'information design', considering a platform that sells the best product at personalised price to those that would purchase inferior qualities. Our results are diverse; evaluating the industries were the platform operates in is crucial. We conclude that the platform benefits consumers whenever it doesn't lead to the formation of a monopoly; however, if the industry would operate under Cournot competition, the platform is always beneficial.

While platform intermediation isn't extensively present in reality, our results extrapolate to the more common case of a data broker intermediary and suggest that the creation of a platform that reduces quality under-provision can benefit consumers. For instance, a bank such as Revolut could arrive at agreements with firms to personalise the cashback (i.e. discounts) it offers through its app, leveraging from the use of bank transaction data - without the need to share it. If so, our results would apply to real-world situations of interest in policy debates. To develop the example, consider the new Apple Virtual Reality Glasses that will sell for \$3,500, competing with Meta's product priced at \$500 and a number of other lower-quality firms. As Apple's product is the new flagship, it has a large mark-up which leaves many consumers unserved. Revolut could use its data to, accounting for costs, sell Apple's glasses at a discount to those that would have to buy from inferior brands. More precisely, Revolut could drive all of the lower quality firms out of the market (except Meta, if competition is a-la-Bertrand) and/or prevent new competitors to enter with lower quality versions, and this would imply a positive gain in consumer surplus.

This story brings to attention the main limitation of the body of work we present; a complete account of the effects of platform intermediation must model firms' innovation investments (i.e. endogenise quality choice). That is, our model should be embedded in a endogenous growth setting that can evaluate how more discrimination alters incentives to innovate. The literature on market structure and innovation is intricate. While greater competition encourages firm investment to differentiate from rivals close in the quality space (Arrow effect), it discourages investment by reducing profits when other firms catch-up (Schumpeter effect). Intuitively, a greater ability to price discriminates will increase the incentive for top firms to innovate - they can reap higher profits by selling their product to a larger mass of consumers. In addition, a smaller number of competing firms reduces technological spillovers, enlarging the incentive of the top firm to innovate (greater Schumpeterian effect). Nonetheless, if more discrimination implies a larger gap between the top and the bottom firm(s), this deters the latter from investing (smaller Arrow effect). As this brief discussion suggests, further research should consider how platform intermediation influences innovation incentives.

A further limitation of our work is the chosen preference specification. Many products have both vertical and horizontal attributes. This merely suggest that determining consumer surplus gains from platform discrimination is also an empirical, rather than strictly theoretical, question. Yet, the body of work we present can provide a base from which to construct structural models. At last, the assumption of perfect information is strong; further work could study how relaxing away from this benchmark affects results. These extensions bring us closer to quantify how much inefficiencies would a platform realistically reduce. In general, our work suggests that information technologies can lead to discrimination that benefit both producers and consumers; proposals that limit the application of new discrimination technologies must be taken with care.

IX. Bibliography

Aguirre, I., Cowan, S. and Vickers, J. (2010). Monopoly Price Discrimination and Demand Curvature. *American Economic Review*, 100(4), pp.1601–1615.

Armstrong, M. (2006). *Price discrimination.* University Library of Munich, MPRA Paper.

Armstrong, M. (2016). Nonlinear Pricing. *Annual Review of Economics*, 8(1), pp.583–614.

Bergemann, D., Brooks, B. and Morris, S. (2015). The Limits of Price Discrimination. *American Economic Review*, 105(3), pp.921–957.

Bergemann, D. and Morris, S. (2019). Information Design: A Unified Perspective. *Journal of Economic Literature*, 57(1), pp.44–95.

Corts, K.S. (1998). Third-Degree Price Discrimination in Oligopoly: All-Out Competition and Strategic Commitment. *The RAND Journal of Economics*, 29(2), p. 306.

Elliott, M., Galeotti, A., Koh, A. and Li, W. (2021). *Market Segmentation Through Information*. mimeo.

Fudenberg, D. and Villas-Boas, M. (2006). *Behavior-Based Price Discrimination and Customer Recognition*.

Gabszewicz, J.J., Marini, M.A. and Tarola, O. (2016). Core existence in vertically differentiated markets. *Economics Letters*, 149, pp.28–32.

Haghpanah, N. and Siegel, R. (2022). Pareto Improving Segmentation of Multi-Product Markets. *Journal of Political Economy*.

Hidir, S. and Vellodi, N. (2020). Privacy, Personalization and Price Discrimination. *Journal of the European Economic Association*.

Ichihashi, S. (2020). Online Privacy and Information Disclosure by Consumers. *American Economic Review*, 110(2), pp.569–595.

Jaskold Gabszewicz, J., Shaked, A., Sutton, J. and Thisse, J.-F. (1986). Segmenting the market: The monopolist's optimal product mix. *Journal of Economic Theory*, 39(2), pp. 273–289.

Johnson, J.P. and Myatt, D.P. (2003). Multiproduct Quality Competition:Fighting Brands and Product Line Pruning. *American Economic Review*, 93(3), pp.748–774.

Maskin, E. and Riley, J. (1984). Monopoly with Incomplete Information. *The RAND Journal of Economics*, 15(2), pp.171–196.

Mussa, M. and Rosen, S. (1978). Monopoly and product quality. *Journal of Economic Theory*, 18(2), pp.301–317.

Pigou, A.C. (1920). *The economics of welfare*. London: Macmillan.

Rhodes, A. and Zhou, J. (2022). Personalized Pricing and Competition. mimeo.

Robinson, J. (1931). *The Economics of Imperfect Competition*. Springer.

Shaked, A. and Sutton, J. (1982). Relaxing Price Competition Through Product Differentiation. *The Review of Economic Studies*, 49(1), p.3.

Stokey, N.L. (1979). Intertemporal Price Discrimination. *The Quarterly Journal of Economics*, 93(3), p.355.

Stole, L.A. (2007). *Price Discrimination and Competition*. Chapter 34 , Handbook of Industrial Organisation, Elsevier.

Thisse, J.-F. and Vives, X. (1988). On The Strategic Choice of Spatial Price Policy. *The American Economic Review*, 78(1), pp.122–137.

Tirole, J. (1988). *The theory of industrial organization*. Cambridge, Mass.: Mit Press.

Varian, H.R. (1989). *Price discrimination*. Chapter 10 , Handbook of Industrial Organization, Elsevier.

X. Appendix 1

$$\underline{\theta} = 0.9, \bar{\theta} = 1.9, \theta_1^d = 14/15, p_1 = \frac{1}{30}(s_2 - s_1), p_2 = \frac{29}{30}(s_2 - s_1)$$

$$C_s^d = \int_{\frac{9}{10}}^{\frac{14}{15}} (s_1\theta - p_1)d\theta + \int_{\frac{14}{15}}^{\frac{9}{10}} (s_2\theta - p_2)d\theta$$

$$C_s^d = \frac{s_1}{2}(\frac{14}{15})^2 - \frac{s_1}{2}(\frac{9}{15})^2 + \frac{s_2}{2}(\frac{19}{15})^2 - \frac{s_2}{2}(\frac{14}{15})^2 - (\frac{14}{15} - \frac{9}{10})^2(s_2 - s_1) - (\frac{19}{10} - \frac{9}{10})^2(s_2 - s_1)$$

$$C_s^d = \frac{11}{360}s_1 + \frac{493}{180}s_2 - \frac{421}{430}(s_2 - s_1)$$

$$p^m = \frac{9}{20}s_2, \theta_0^m = \frac{9}{20}$$

$$C_s^m = \int_{\frac{9}{10}}^{\frac{19}{10}} s_2\theta \ d\theta \ - (\bar{\theta} - \theta_0)p$$

$$C_s^m = \frac{s_2}{2}(\frac{19}{10})^2 - \frac{s_2}{2}(\frac{9}{19})^2 - \frac{361}{400}s_2$$

$$C_s^m = \frac{759}{400}s_2$$

$$C_s^d > C_s^m -> \frac{11}{360}s_1 + \frac{493}{180}s_2 - \frac{421}{430}(s_2 - s_1) > \frac{759}{400}s_2$$

$$-> \frac{11}{360}s_1 + \frac{3029}{3600}s_2 - \frac{421}{430}(s_2 - s_1) > 0 -> \frac{1739}{1800}s_1 > \frac{113}{1200}s_2$$

XI. Appendix 2

$$\theta_1 = \frac{2\bar{\theta} - \underline{\theta}}{3} - \frac{\bar{\theta} - 2\underline{\theta}}{3} = \frac{2\underline{\theta} + 1}{3}, p_1 = \frac{\bar{\theta} - 2\underline{\theta}}{3}(s_2 - s_1) = \frac{1 - \underline{\theta}}{3}(s_2 - s_1)$$
$$p_2 = \frac{2\bar{\theta} - \underline{\theta}}{3}(s_2 - s_1) = \frac{2 + \underline{\theta}}{3}(s_2 - s_1)$$

$$C_s^d = \int_{\underline{\theta}}^{\frac{2\underline{\theta}+1}{3}} (s_1\theta - p_1)d\theta + \int_{\frac{2\underline{\theta}+1}{3}}^{\underline{\theta}+1} (s_2\theta - p_2)d\theta$$

$$C_s^d = \frac{s_1}{2}(\frac{2\underline{\theta} + 1}{3})^2 - \frac{s_1}{2}\underline{\theta}^2 + \frac{s_2}{2}(\underline{\theta} + 1)^2 - \frac{s_2}{2}(\frac{2\underline{\theta} + 1}{3})^2 - ((\frac{1 - \underline{\theta}}{3})^2 - (\frac{2 + \underline{\theta}}{3})^2)(s_2 - s_1)$$

$$C_s^d = \frac{s_1}{18}(4\underline{\theta}^2 + 4\underline{\theta} + 1) - \frac{s_1}{2}\underline{\theta}^2 + \frac{s_2}{2}(\underline{\theta}^2 + 4\underline{\theta} + 1) - \frac{s_2}{18}(4\underline{\theta}^2 + 4\underline{\theta} + 1) - \frac{1}{9}(2\underline{\theta}^2 - 2\underline{\theta} + 5)$$

$$C_s^d = \frac{s_2}{18}(\underline{\theta}^2 + 10\underline{\theta} - 2) + \frac{s_1}{18}(8\underline{\theta} + 11\underline{\theta} - \underline{\theta}^2)$$

$$p^m = \frac{\bar{\theta}s}{2} = \frac{(\underline{\theta} + 1)s_2}{2}, \theta_0^m = \frac{\underline{\theta} + 1}{2}$$

$$C_s^m = \int_{\underline{\theta}}^{\bar{\theta}} s_2\theta \ d\theta \ - (\bar{\theta} - \theta_0)p$$

$$C_s^m = \frac{s_2}{2}(\underline{\theta} + 1)^2 - \frac{s_2}{2}\underline{\theta}^2 - \frac{s_2}{4}(\underline{\theta} + 1)^2$$

$$C_s^m = \frac{s_2}{4}(2\underline{\theta} + 1 - \underline{\theta}^2)$$

$$C_s^d > C_s^m -> \frac{s_2}{18}(\underline{\theta}^2 + 10\underline{\theta} - 2) + \frac{s_1}{18}(8\underline{\theta} + 11\underline{\theta} - \underline{\theta}^2) > \frac{s_2}{4}(2\underline{\theta} + 1 - \underline{\theta}^2)$$

$$-> s_1 > s_2\frac{13 - 2\underline{\theta} - 11\underline{\theta}^2}{22 + 16\underline{\theta} - 2\underline{\theta}^2}$$