Towards a Network Économique

Creating a network that represents society's economic value flows.

Introduction

We can imagine our social reality as an immense and infinitely complex system of value streams. Countless fast-flowing rivers connect individuals - which themselves all produce, exchange and consume value. A variety of intricate processes, guided by our institutional arrangement, shape the rivers' channels. In our capitalist atmosphere, most of the value we consume is commodified, and thus takes an economic form. This essay aspires to comprise all of society's economic value flows in, what we shall call, a Network Économique.

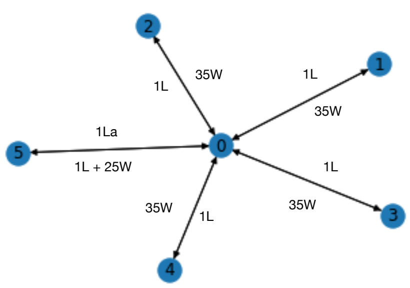
The essay's structure is as follows. First, we will sketch a brief story that portrays the main motivations behind the essay. A foundation section will continue, which will lay out the grounding assumptions and mathematical tools used along the essay. Transitioning into the main body, we'll present a basic a modelling framework capable of simulating exchange in a society. All will converge in the formalisation of the IF matrix - the main contribution of the essay. At last, we will represent our IF matrix as a Network Économique, and formulate a series of micro-macro measures for its analysis. In between sections, examples will accompany our theory.

Motivation

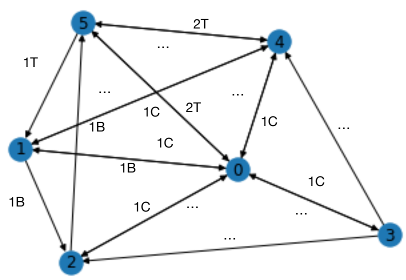
To reflect the purpose behind the Network Économique, and thus get a broad picture of what the essay will attempt, it is worth presenting an illustrative example. Let's imagine a society, A, that lives in an as-if feudalist structure. An example founded on a real case study would be more appropriate, but given time constraints, we shall depict our society through the following fictitious narrative:

A) Society A is home to 6 individuals (**I**), though dominated by **I**0. Driven by selfishness, **I**0 employs his ("1") **M**achine to produce deadly **A**rmament. As a result of **I**0's military power, he possesses all ("6") of the fertile **L**and in our made-up universe. Besides, society A is pretty prosaic. **I**1, **I**2, **I**3 and **I**4 rent "1" of **L**and from **I**0 for "35" of **W**heat. They then use the **L**and to each produce "40" of **W**heat. The last member of society A, **I**5, is a close partner of **I**0. In exchange of "1" of **L**and and "25" of **W**heat, **I**5 sells "1" of his skilled **La**bour to **I**0, which **I**0 uses to make "50" **A**rmament.

In our story, we have indirectly described the institutional structure of a society, while displaying how value flows between its individuals. We can graph the economy of such a society with a Network Économique based on the narrative. Indeed, many events can change a society's distribution of value. For example, a revolution against **I**0 in A could liberalise the available **L**and and **M**achinery, giving rise to society B.



An example of A's potential representation. Here, W stands for Wheat, L for Land etc.



An example of B's potential representation. Here, T stands for Telephone, C for Chair etc.

As we see, our Network Économique has the potential of providing a superficial description of a society's structure at a specific point in time. Ideally, formulating network measures can help us understand the characteristics of a particular Network Économique. Although the societies presented in the examples are radially different from our own, obtaining the necessary information is the only barrier preventing us from creating a realistic Network Économique.

Foundations: Value and Information Matrices

As in Marx's Capital, the starting point of our inquiry is the conceptualisation of 'value'. Characterising 'value' is a primary topic in economic philosophy, as any consistent economic model must employ an underlying theory of value. But, what makes 'value' so supreme? Well, we could allege that the purpose of life itself is 'experiencing value'. Such a powerful statement is the prevalent base of economic theory - which poses individuals as rational utility maximisers. This postulate is, arguably, incomplete. For example, it ignores questioning the source of value. Right away, we can notice that 'value' is a remarkably abstract substance - which makes a precise definition of it is somewhat problematic. We certainly know that 'value' exists and can be identified in the many ways that it materialises around us. For this essay, we will assume that:

(i) Value can be categorised (i.e. takes a range of forms) into use-value types (eg. food, clothing, machinery, labour)

(ii) Value originates from primitive use-value types (or factors of production)

(iii) Production Processes, where a production function transforms primitive use-value types into finished use-value types (or goods), augment the character of value

(iv) Consumption Processes destroy value by exploiting finished use-value types. The resultant of consumption is 'utility' (i.e. experienced value)

(v) Humanity aspires to reach higher utility levels i.e. to consume as much experienced value as possible

Essentially, value is not unlimited; individuals must create it from primitive resources for its later consumption. What is of our interest is; how can we account for information which regards value? We can store information about value, i.e. the outcome of a process at a period, using a matrix. The following matrices naturally arise from the previous discussion:

(viii) We illustrate the total value created by society through a Production Outcome, P, matrix. This matrix represents the total value available for social exchange and the individual who owns it.*

(ix) We illustrate the total value destroyed by society through a Consumption Outcome, C, matrix. This matrix represents the total value consumed by a society and who has experienced it.*

$$P*_{i,uvt} = \begin{pmatrix} p_{1,a} & p_{1,b} & \cdots & p_{1,uvt} \\ p_{2,a} & p_{2,b} & \cdots & p_{2,uvt} \\ \vdots & \vdots & \ddots & \vdots \\ p_{i,a} & p_{i,b} & \cdots & p_{i,uvt} \end{pmatrix} \quad C*_{i,uvt} = \begin{pmatrix} c_{1,a} & c_{1,b} & \cdots & c_{1,uvt} \\ c_{2,a} & c_{2,b} & \cdots & c_{2,uvt} \\ \vdots & \vdots & \ddots & \vdots \\ c_{i,a} & c_{i,b} & \cdots & c_{i,uvt} \end{pmatrix}$$

"i" stands for individual,"uvt" stands for use-value type

These matrices are highly informative and a convenient way to summarise relevant economic information. Moreover, they open a question: what lies between production and consumption? Assuming individuals specialise in producing a limited amount of use-value types but desire to consume most of them, individuals will engage in exchange. Hence:

(x) We illustrate the social value exchange in a period, using an Exchange, E, matrix. This matrix represents the quantity of a use-value types that an individual has gained or lost after exchange takes place

$$E_{i,uvt} = \begin{pmatrix} e_{1,a} & e_{1,b} & \cdots & e_{1,uvt} \\ e_{2,a} & e_{2,b} & \cdots & e_{2,uvt} \\ \vdots & \vdots & \ddots & \vdots \\ e_{i,a} & e_{i,b} & \cdots & e_{i,uvt} \end{pmatrix}$$

In a world in which individuals are purely self-sufficient, the concept of value would hold no riddle. The truth is that other individuals produce most of the value that we consume. A direct implication of this is the need for a method by which a community can precisely measure value. Quantifying value is important for our essay, so we need to assume that:

(xi) *We can measure value using 2 units: Raw (eg. 1 machine) and Valorised ($1 of machinery)*

(xii) *Valorisation implies that we can compare different use-value through a homogeneous constant. Thus, a numeraire, i.e. $1, universally describes what exactly is 1 unit of value eg. 'x' of machinery = 'y' of wheat = $1*

(xiii) *Valorisation takes place at a social level i.e. PV is equal for all individuals, and all individuals perceive $1 of value equally*

Again, we can use matrices to store information regarding valorisation:

(xiv) *We illustrate the valorised ($) respective of a unit of a raw use-value type using a Price Vector, PV, matrix.*

$$PV_{uvt} = \begin{pmatrix} p_a \\ p_b \\ \vdots \\ p_{uvt} \end{pmatrix}$$

What determines valorisation is beyond this essay - it could be a function that valorises every use-value type owned by an individual. In real life, we mainly rely on markets (or equilibrium market prices) to solve this problem.

Notice that the P*, C*, E and PV matrices will be interconnected in a variety of ways. For there being no inconsistencies in our analysis, we shall assume that:

- $C* = P* - E$: exchange is the difference between one's production and consumption.
- $P*xPV = C*xPV$: the total value consumed by an individual must equal to the one he produces.
- $ExPV = 0$: the total value exchanged in society must be equal to 0.

In general, the following conditions imply that from P* and E, we can easily calculate the values of C* and PV. This will be both useful and important for the remaining of our essay. Beyond, an implication of the conditions is that exchange is double sided. So, all positive values of the E matrix, eg. $E_{1,W} = 1$, must be accompanied by a negative one eg. $E_{0,W} = -1$. Why? Simply because if an individual owns 'x' more of a use-value type after exchange, another individual must have faced the cost of owning 'x' less of it.

<u>Application 1:</u> Information Matrices for Example A

We can apply the previous abstract theory to summarise the story of Example A. In this particular universe, there is a defined set of use-value types (**M**achines, **A**rmament, **L**and, **W**heat, **La**bour). Some of these use-value types are primitive (**M**achines, **L**and, **La**bour), while others are finished (**W**heat, **A**rmament). In reality, some use-value types could can be categorise both as primitive or finished, although this isn't important for our analysis. For example, **L**and can be used as a factor of production (eg. For the production of **W**heat), or directly consumed. Using information matrices, and accounting for the processes described in the story, we obtain the following:

$$P*_{[0,1,2,3,4,5],[L,M,A,W,La]} = \begin{pmatrix} 6 & 6 & 50 & 0 & 0 \\ 0 & 0 & 0 & 40 & 0 \\ 0 & 0 & 0 & 40 & 0 \\ 0 & 0 & 0 & 40 & 0 \\ 0 & 0 & 0 & 40 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad E_{[0,1,2,3,4,5],[L,M,A,W,La]} = \begin{pmatrix} -5 & 0 & 0 & 120 & 1 \\ 1 & 0 & 0 & -35 & 0 \\ 1 & 0 & 0 & -35 & 0 \\ 1 & 0 & 0 & -35 & 0 \\ 1 & 0 & 0 & -35 & 0 \\ 1 & 0 & 0 & 20 & -1 \end{pmatrix}$$

Our story only captures information about the outcomes of production and exchange. From this, we can easily calculate what each individual ends consuming, and the exchange prices (i.e. social valorisation) of each use-value type.

$$C*_{[0,1,2,3,4,5],[L,M,A,W,La]} = P* - E = \begin{pmatrix} 6 & 6 & 50 & 0 & 0 \\ 0 & 0 & 0 & 40 & 0 \\ 0 & 0 & 0 & 40 & 0 \\ 0 & 0 & 0 & 40 & 0 \\ 0 & 0 & 0 & 40 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} -5 & 0 & 0 & 120 & 1 \\ 1 & 0 & 0 & -35 & 0 \\ 1 & 0 & 0 & -35 & 0 \\ 1 & 0 & 0 & -35 & 0 \\ 1 & 0 & 0 & -35 & 0 \\ 1 & 0 & 0 & 20 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 6 & 50 & 120 & 1 \\ 1 & 0 & 0 & 5 & 0 \\ 1 & 0 & 0 & 5 & 0 \\ 1 & 0 & 0 & 5 & 0 \\ 1 & 0 & 0 & 5 & 0 \\ 1 & 0 & 0 & 20 & 0 \end{pmatrix}$$

$$P*xPV = \begin{pmatrix} 6a + 6b + 50c \\ 40d \\ 40d \\ 40d \\ 40d \\ e \end{pmatrix} = C*xPV = \begin{pmatrix} a + 6b + 50c + 120d + e \\ a + 5d \\ a + 5d \\ a + 5d \\ a + 5d \\ a + 20d \end{pmatrix}$$, letting d = 1 and solving we obtain: $PV_{[L,M,A,W,La]} = \begin{pmatrix} 45 \\ 0 \\ 0 \\ 1 \\ 65 \end{pmatrix}$

How can we interpret the results?
Focusing in C*, we see that the consumption of **W**heat and **A**rmament is highly concentrated on **I**0. As the Labour of **I**5 is valuable for **I**0, **I**5 is able to consume "15" (or $15) more of **W**heat than the other "peasant" individuals.
The PV also contains interesting insights. Notice that **M** and **A** are not valorised (i.e. they are 0). Why? As **I**0 produces and consumes the whole of **M** and **A**, i.e. they are not involved in Exchange, valorising them purely through exchange values is not possible. In contrast, from the exchange interactions of **L**, **La** and **W**, we can calculate the rate of exchange (or price) that each of them respectively hold - hence their social valorisation. From this, we infer that the value of **La** > **L** > **W**.

<u>Process Modelling:</u> Economic Processes, Complexity and Modelling

Until this point, we have been working with pre-defined information extracted from stories. In other words, we have partially ignored what occurs in-between information matrices. In reality, each of our defined matrices is the result of a series of complex processes which take place inside our social institutions. Modelling these (mainly microeconomic) processes is beyond this project. Nevertheless, if we wan't to go beyond representing stories, we shall create a sufficiently realistic modelling framework so that we can capture interesting social properties in our analysis.

Our main question is; what processes determine P*, C*, E and PV? We must account for four fundamental processes:
· Transformation of value - To obtain P* we need to model how individuals decide to employ primitive use-value types to produce finished use-value types.
· Valorisation - To obtain PV we need to model how prices are determined in an economy.
· Exchange - To obtain E and C* we need to model how individuals decide upon what to consume
· Institutional Setting - We have to constantly account for the constraints that structure economic interactions eg. Can individuals purchase others' primitive use-value types (eg. labour)? eg.2 Are there power relations in our society?

The processes above are deeply interrelated eg. valorisation will affect the way individuals decide what to produce. Thus, in a rigorous approach, we would calculate the value of all matrices endogenously, for a given institutional setting. Furthermore, we would rely on sophisticated economic modelling, such as general equilibrium and agent-based models. It becomes apparent that whenever we try to explain the real world, we have to account for the trade-off between realism and simplicity.

For this essay, we will model simple economic processes to generate C* and E from some primary inputs (P*, PV and consumption preferences). The following framework is employed:

1) We set PV and P* exogenously, assuming primitive use-value types can be exchanged, and thus are included in P*
2) We set consumption preferences (i.e. consumption coefficients for each use-value type) exogenously, accounting for their relation to PV and assume reasonable behavioural assumptions
3) We calculate each individual's income based on P* and PV, interpreting this as an individuals' credit for consumption

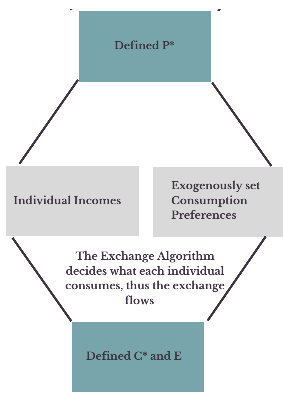


Figure 1

4) We use an 'Exchange Algorithm' that generates an E, thus C*, given the inputs on individual incomes (2) and consumption preferences (3).

This exact modelling process is summarised in Figure (1). Note that the Exchange Algorithm simulates how individuals use their defined income to purchase the use-value type that holds a higher consumption coefficient. This is a simple way to model how individuals will consume, so it will indirectly define exchange in a society. The modelling is carried by a Python code. It is important to note that some further assumptions, which are flexible, will be embedded in the inputs that the code takes.

Our modelling approach has imperfections and isn't entirely realistic. A more precise framework, capable of determining PV, E and C* endogenously, is formalised by Cheng and Wellman (1998). They employ an iterative process that calculates competitive equilibria from individual single-good demand functions - what they call 'The Walras Algorithm'. Regardless, our framework can deliver sufficiently meaningful information matrices for our analysis.

Application 2: Modelling Society C

Using our modelling framework, we now proceed to simulate society C, which is home to 6**I's** and has a predefined set of existing use-value types (**L**and, Weaving **M**achinery, **C**lothing and **W**heat). (Note that **L**abour is not included as a use-value type, as we assume it will won't be exchanged) The following primary inputs are given:

How can we interpret the inputs?

$$PV_{[L,M,W,C]} = \begin{pmatrix} 20 \\ 10 \\ 1 \\ 2 \end{pmatrix} \qquad P^*_{[0,1,2,3,4,5],[L,M,W,C]} = \begin{pmatrix} 3 & 2 & 0 & 0 \\ 0 & 0 & 40 & 0 \\ 0 & 0 & 40 & 0 \\ 0 & 0 & 40 & 0 \\ 0 & 0 & 0 & 20 \\ 0 & 0 & 0 & 20 \end{pmatrix} \qquad CC_{[0,1,2,3,4,5],[L,M,W,C]} = \begin{pmatrix} 0 & 0 & 100 & 110 \\ 1000 & 0 & 110 & 100 \\ 1000 & 0 & 110 & 100 \\ 1000 & 0 & 110 & 100 \\ 0 & 1000 & 110 & 100 \\ 0 & 1000 & 110 & 100 \end{pmatrix}$$

We assume L is a factor of production of W, and that M is a factor of production of C

We assume that each time a unit of a use-value type is purchased, its consumption coefficient decreases by 1 (decreasing marginal utility)

We see that the value of **L** > **M** > **C** > **W**. Furthermore, P* suggest there is a high degree of specialisation. Consumption coefficients require some deeper explanation. The highest coefficient (1000) is given to those primitive use-value types needed by individuals in production. Besides, individuals first use their incomes to consume up to 10 units of wheat (say, for survival). Afterwards, they will alternate between purchasing clothing and wheat.

Our code will first calculate the anticipated income of each individual, and use income as 'credits' for consumption to mechanically calculate the following output.

$$I_{[0,1,2,3,4,5]} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad C^*_{[0,1,2,3,4,5],[L,M,W,C]} = \begin{pmatrix} 0 & 0 & 46 & 17 \\ 1 & 0 & 14 & 3 \\ 1 & 0 & 14 & 3 \\ 1 & 0 & 14 & 3 \\ 0 & 1 & 16 & 7 \\ 0 & 1 & 16 & 7 \end{pmatrix} \qquad E_{[0,1,2,3,4,5],[L,M,C,W]} = \begin{pmatrix} -3 & -2 & 46 & 17 \\ 1 & 0 & -26 & 3 \\ 1 & 0 & -26 & 3 \\ 1 & 0 & -26 & 3 \\ 0 & 1 & 16 & -13 \\ 0 & 1 & 16 & -13 \end{pmatrix}$$

Flow Matrix: A Network Representation of our Economy

As of the preceding section, we have generated meaningful information about a society's economy using a basic modelling process. We shall proceed to present our information matrices as conveniently as possible. This involves explicitly asking: what matrix can best represent an economy? This essay puts forward the IF matrix as a humble attempt to answer the previous question. Essentially, the IF matrix captures the value flows between individuals after all economic processes have taken place. Breaking down the IF matrix, we should first highlight that both axes take an individual as its unit. A row of the IF matrix captures the value received by an individual, r, from another individual, s. For example,$IF_{1,s}$stores how much value **I**1 has received from **I**1, **I**2… **I**s. Which leaves a column storing the value sent by an individual, s, to another individual, r. For example, $IF_{r,2}$ stores how much value individual 2 sends to , **I**1, **I**2… **I**r.

$$IF_{r,s} = \begin{pmatrix} f_{1,1} & f_{1,2} & \cdots & f_{1,s} \\ f_{2,1} & f_{2,2} & \cdots & f_{2,s} \\ \vdots & \vdots & \ddots & \vdots \\ f_{r,1} & f_{i,1} & \cdots & f_{r,s} \end{pmatrix} \qquad IF_{1,s} = \begin{pmatrix} f_{1,1} & f_{1,2} & \cdots & f_{1,s} \end{pmatrix} \qquad IF_{r,2} = \begin{pmatrix} f_{1,2} \\ f_{2,2} \\ \vdots \\ f_{r,2} \end{pmatrix}$$

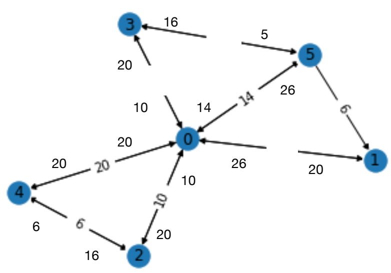
But, how does the IF matrix relate to our P*, C*, E and PV matrices? It turns that the IF matrix is just an alternative way to arrange, or rather compress, the information expressed by P*, C*, E and PV. We can illustrate our point through an example. Let's imagine that **I**1 produces '2' of **W**heat, while **I**0 produces '0' of **W**heat. This would be expressed in a P* matrix as $P_{0,W} = 0$ and $P_{1,W} = 2$. For some undetermined reason, there is a flow, or transfer, of '1' of **W**heat from **I**1 to **I**0, which means that $E_{0,W} = 1$ and $E_{1,W} = -1$. This means that, following the conditions in the foundations section, we are left with $C_{0,W} = 1$ and $C_{1,W} = 1$. We can summarise all of the above information elegantly in the IF matrix as $IF_{1,0} = 1$(assuming,$PV_{[W]} = 1$) - which simply means that **I**0 receives $1 of value from **I**0. Notice that the IF matrix can't be derived through conventional matrix operations from our original information matrices. Nevertheless, we can trace down all of the information presented in the IF matrix to its respective values in the P*, C* and E matrices.

Formally deriving the IF matrix is a mechanical process. Recall that for every $E_{i,uvt} = +x$ store in our E matrix, there is a $E_{i,uvt} = -x$ counter. From this, we can record which individual is gaining that use-value type (i.e. the receiver) and which individuals is loosing that use-value type (i.e. the sender) for each exchange transaction. We can rename the individual loosing value as "s", and the one gaining value as "r" (eg. s= **I**1 and r = **I**0). In addition, we also record the positive quantity of value transferred, 'x' (eg. x = $1 = 1W). Only after collecting this from our original information matrices, we can express it as $IF_{r,s} = x$. Repeating this process for all of the information in the P*, C*, E and PV matrices leads us to the IF matrix. The interpretation of the IF matrix is straightforward; it is an unconventional way to picture an economy.

Application 3: Representing the IF Matrix

A Network Économique is the graphical representation of an IF matrix. Our code can generate the IF matrix of society C, and its corresponding Network Économique:

$$IF_{[0,1,2,3,4,5],[0,1,2,3,4,5]} = \begin{pmatrix} 0 & 20 & 20 & 20 & 10 & 10 \\ 26 & 14 & 0 & 0 & 0 & 0 \\ 10 & 0 & 14 & 0 & 16 & 0 \\ 10 & 0 & 0 & 14 & 0 & 16 \\ 20 & 0 & 6 & 0 & 14 & 0 \\ 14 & 6 & 0 & 6 & 0 & 14 \end{pmatrix}$$

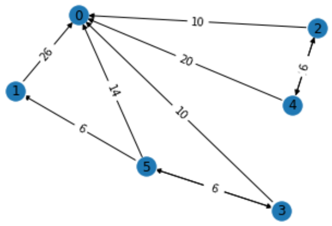


How can we interpret the results?

It becomes apparent that all individuals heavily interact with I0, which faces both a large quantity of inflowing and outflowing value. We also observe that a pair (I2 and I4) and a triplet (I3 , I5 and I1) of interactions arises. The diagonal of the IF matrix also contains some numbers, which means that some of the value produced by an individual is kept for their own consumption - technically, this is not part of an economy. This occurs as this is a small, simple economy and our modelling allows people to use their incomes to buy their own produce (which could be modified)

Some individuals use their income to by primitive use-value types, such as land, which doesn't translate to a direct finished use-value type consumption. So, we can adapt our IF matrix to only record the flows of finished use-value types - which are the ones through which utility is obtained.

$$IF_{[0,1,2,3,4,5],[0,1,2,3,4,5]} = \begin{pmatrix} 0 & 20 & 20 & 20 & 10 & 10 \\ 0 & 14 & 0 & 0 & 0 & 0 \\ 0 & 0 & 14 & 0 & 16 & 0 \\ 0 & 0 & 0 & 14 & 0 & 16 \\ 0 & 0 & 6 & 0 & 14 & 0 \\ 0 & 6 & 0 & 6 & 0 & 14 \end{pmatrix}$$



<u>Network Économique Measures:</u> Micro and Macro Measures

Given a defined Network Economique, we now ask: what features of a society can we analytically observe? For example, we might question whether there is a high number of economic interactions in a society. Directly answering such a question is challenging. Most often, we must account for a wide variety of factors that can affect our answer. Formulating micro and macro network measures will help us understand some of the characteristics of a society's economy. Generally, our Network Économique will take the form of a weighted directed graph, which will require adapting some of the conventional network measures.

(1) <u>Macro Measure:</u> Density

By macro measures, we refer to tools which highlight properties about an entire network. The density of a Network Économique is a proxy of the number of economic interactions that arise between individuals in a society. To facilitate the measurement of density, we shall transform the IF matrix into an unweighted (U) IF matrix. An entry in the UIF matrix will capture if the exchange relationship between two individuals is 'strong' or 'weak'. So, for each $IF_{r,s} = x$ in our IF matrix, whenever $x > iI_r$, we shall redefine the interaction between "r" and "s" as strong (i.e. $UIF_{r,s} = 1$) . Otherwise, if $x < iI_r$, the relationship will be seen as weak (i.e. $UIF_{r,s} = 0$). Here, I_r is the income of individual "r", and "i", where $i \in [0,1]$, is the threshold establishing what share of an individual's total income must be involved so that an interaction is considered as being relevant. For example, if we set $i = 0.01$, most interactions between individuals will be recorded as strong. So, deciding the 'i' parameter will determine our density measurements.

We will compute density using the formula provided in the Networkx Python library:

$$d = \frac{m}{n(n-1)}$$

"n" stands for the number of nodes, and "m" for the number of edges of our IF

The intuition behind our density measure can be shown via some extreme scenarios. In a society where all individuals are completely self-sufficient, so there are no value flows between individuals, density will be 0. The contrary can also occur, when all individuals are highly specialised in the production of a unique use-value type, and individuals demand all types of use-value type. So, we expect specialisation to increase our Network Économique's density.

(2) Micro Measures: Centrality

By micro measures, we refer to tools which highlight a property about a specific node in a network. The centrality of a node can reflect the importance (or power) of an individual in an economy. A standard way to analyse centrality in weighted network is by adding the weights of a node's edges. The following formula is used (Agneessens, Opsahl and Skvoretz, 2010):

$$s_i = C_{\mathrm{D}}^{w}(i) = \sum_{j}^{N} w_{ij}$$

This weighted centrality measure will simply rank individuals by their incomes. So, this is just an alternative way to observe inequality in an economy. A more insightful approach is that provided by Agneessens et al. (2020), which generalises node centrality in a weighted network as a combination of both the number of edges and the edges' weights. They provide the following formula:

$$C_{\mathrm{D}}^{w\alpha}(i) = k_i \times \left(\frac{s_i}{k_i}\right)^{\alpha} = k_i^{(1-\alpha)} \times s_i^{\alpha}$$

By combining both degree and strength, we can capture in a more realistic way how central an individual is in an economy. For example, an individual with a strong total weight but low degree, (imagine it interacting heavily with only with one individual), will receive a similar score to an individual with a lower total weight but a high degree (imagine it having small value flows with a high amount of individuals). Obviously, an individual with a high weight and high degree will be regarded by our ranking as an important and powerful agent of an economy. This can suggest, among other things, that his decisions will likely affect a large number of individuals, and that he is a highly active member of a society.

(3) Potential Advanced Measures

We are limited by the complexity of our Network Économique when applying network measures - further developing it would allow us to capture more complex phenomenon with our network measures. So, a continuation of the project would consider the following questions:

(iv) Do individuals with a high income interact more with individuals that also have high incomes?

This could be examined via Homophily measures. For example, we could divide individuals into groups with shared characteristics (eg. 3 node types: high, medium and low incomes), and measure if there is a correlation between the size of interactions and income types.

(v) Should individuals that interact with other influencing individuals be considered as more central?

This could be examined via Eigen-vector centrality - which accounts for network feedback effect within individuals. Potentially, an individual that has a low income but closely interacts with a powerful individual, should have a higher influence in an economy than one that has a low income and is poorly connected.

(vi) How does a economic negative/positive shock disperse through the Network Économique

Potentially, we could apply a betweenness centrality measure to analyse if an individual (or an industry) acts is a bridge (i.e. strategically located node) in an economy. Hypothetically, this could be used as a proxy to test if some individuals are vital for the provision of economic stability of a society.

Conclusion and Evaluation

This essay presents a preliminary version of a framework that could expand into a realistic representation of our economy. An advantage the methodology employed is that information matrices are a flexible and encompassing way to summarise the economic life of a society. Furthermore, our modelling allows us to track how modifying certain parameters affects the big picture, and has extensive consequences across all members in a Network Économique. Overall, our Network Économique allows us to inquire into deep questions from an analytical perspective.

At this point, we can only speculate, but in the future we could answer: is there an ideal Network Économique density? To get into more meaningful applications the roadmap is clear. More sophisticated microeconomic models have to be employed to bring alive our Network Économique. Besides, to observe more complex structural properties of a society, network measures should be for formulated in detail, and be rigorously tested and applied for their better understanding.

Bibliography

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